DEVELOPING STUDENTS' METACOGNITIVE SKILLS IN MATHEMATICS CLASSROOM

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ABSTRACT: Twenty-first century mathematics education is about facing novel real-world problems, nurturing creative thinking skills and cultivating productive ways of learning. In attempting to innovate teaching and learning in order to prepare a new generation for the demands of this new era, many educators have discovered the value of metacognition. This paper presents the importance of metacognition to the learning of mathematics, employed metacognitive skills in the process of solving mathematics problems.

KEYWORDS: Metacognitive skills, higher-order thinking skill, mathematics problem solving, mathematics teaching methods.

1. INTRODUCTION

Mathematics is always one of the difficult subjects for school students ([YK12]); metacognitive processes focus on students' ability monitor and regulate their own cognitive processes employed during problem solving ([AA92, Sch92]). Many scholars have argued that emphasis on cognition without a corresponding emphasis on metacognitive thinking renders a problem-solving endeavour incomplete; Distinguishing between what is cognitive from what is metacognitive has been problematic ([GL85, GG96]). Some experts more specifically define that metacognition is one's knowledge about cognitive processes and one's awareness on a mathematical problem involving the process of planning, monitoring, and evaluation of problem solutions ([Fla79]). Metacognition, the ability to think about thinking, is central in this process as it involves an awareness of the cognitive process and ability to control it. Metacognition has two main components: knowledge about cognition and control/regulation of cognition ([BB84]). According to ([Fla79]), metacognition is a system which organizes information, experiences, objectives and strategies. Metacognition, which means thinking about thinking, generally covers various skills that are inter-related to thinking and learning, which are critical thinking, reflective thinking, problem-solving and making a decision. Individuals, who have more developed metacognitive skills, are also better problem solvers, decision makers and critical thinkers than others. Problem is a case which contains open-ended questions, which attracts the attention of an individual and which the individual does not have the necessary algorithm and method knowledge to solve these questions ([BN91]); Problem solving is to know what should be done when it is not known what to do. Problem solving process, which is only perceived as reaching the correct solution, in fact, covers wider mental process and skills than this. Problem solving is finding a way and getting rid of the difficulty in addition to reaching the solution. ([TT16]), mathematics teachers in their efforts to help their pupils' success, strive to find the best ways to teach, so that the pupils are engaged in the learning process.

In domain of mathematics, solving problems is a routine process. This paper presents the influence of students' metacognitive skills on their success in solving mathematical problems. The research is focused on the study of students' cognitive and metacognitive skills in order to analyze: Is there anything that could be taught that would improve their ability to assemble effective problem.

2. RESEARCH CONTENTS

2.1. Purpose of the study

The main focus of this study is to build a theoretical model of metacognition in Mathematics within the context of problem solving among the junior Mathematics major students of Le Hong Phong hight school, Dong Nai province, Vietnam.

2.2. Research questions

The study sought to provide answers to the following questions:

1. What thinking skills do individual students utilize as they go around thinking about problem solving?
2. What pattern of thinking process can be drawn

relative to the responses?

3. What pattern of metacognition is exhibited by the students in the context of metacognitive functions?
4. What metacognitive model of learning can be abstracted based on the data gathered?

2.3. Methodology

The study employed the survey type of descriptive research. The multi stage sampling technique was used to select the subjects for the study. The tool used was a structured questionnaire. The students in this study were the eighteen study of grade 12 of the Le Hong Phong hight school, Dong Nai province, Vietnam. The class was composed of four males and fourteen females divided into ability grouping namely group 12A is deemed as the higher ability group and group 12B is deemed as the lower ability group based on teachers comments for students.

2.4. Mode of gathering data and data analysis

The students were made to answer the problems in two modes. (1) teacher of gathering data was administering the first two questions to all students at the same time. Then, they were asked to write down their own reflections as to what went on their minds as they were answering the questions. (2) each student was first asked to answer the last two problems and was interviewed to reflect on thinking skills he had while answering the question. The students were just advised by the teachers to describe in sequence what went on in their minds as they were answering the questions. Thereby also the researcher noted the behavior of the students. The interviews conducted were all recorded.

Data analysis: the students were asked to reflect on their thinking as they went around thinking the problems, their metacognitive actions were noted. Transcript of the recorded interviews and written reflections were the basis for determining the thinking skills of the students.

Student thinking skills were considered and matched according to Marzano's definition of thinking skills include defining problem, setting goals, observing, formatting questions, encoding, recalling, comparing, classifying, ordering, representing, identifying, inferring, predicting, elaborating ([M+98]).

Based on the use of these cognitive skills and metacognitive functions shown, a metacognitive model of learning Mathematics in the context of problem solving was abstracted. We categorized into three metacognitive functions as metacognitve awareness (MA), metacognitive evaluation (ME) and metacognitive regulation (MR).

3. RESULTS AND DISCUSSION

This section provides answers to the research questions raised.

Research shows that metacognitive skills can be taught to students to improve their learning ([NS02]). As ([HT14], [Ha15]) state that, meta-cognition refer to "cognition about cognition" or "knowing about knowing". That mean, meta-cognition thinks about one's own thinking process such as study skills. Therefore she analyses its role and application into the education process. memory capabilities, and the ability to monitor learning, meta-cognitions is one of the most important components of mathematics problem solving. Students "construct knowledge" using cognitive strategies, and they guide, regulate, and evaluate their learning using metacognitive strategies. It is through this "thinking about thinking," this use of metacognitive strategies, that real learning occurs. As students become more skilled at using metacognitive strategies, they gain confidence and become more independent as students. "Metacognitive skills" and what one knows about his own cognitive abilities. Metacognition is a process that spans three distinct phases, and that, to be successful thinkers, students must do the following:

(1) Develop a plan before approaching a learning task, such as reading for comprehension or solving a math problem. During the planning phase, students can ask, What am I supposed to learn? What prior knowledge will help me with this task? What should I do first? What should I look for in this reading? How much time do I have to complete this? In what direction do I want my thinking to take me?

(2) Monitor their understanding; use "fix-up" strategies when meaning breaks down. During the monitoring phase, students can ask, How am I doing? Am I on the right track? How should I proceed? What information is important to remember? Should I move in a different direction? Should I adjust the pace because of the difficulty? What can I do if I do not understand?

(3) Evaluate their thinking after completing the task. During the evaluation phase, students can ask, How well did I do? What did I learn? Did I get the results I expected? What could I have done differently? Can I apply this way of thinking to other problems or situations? Is there anything I don't understand any gaps in my knowledge? Do I need to go back through the task to fill in any gaps in understanding? How might I apply this line of thinking to other problems?

The students were found to employ the following thinking skills as they solve the problems: recalling, representing, identifying relations, elaborating, defining the problem, establishing criteria, setting goals, comparing, verifying. Based on the use of these cognitive skills and metacognitive functions shown, a metacognitive model of learning Mathematics in the context of problem solving was abstracted ([Thu16]).

- *Some examples students' metacognitive skills development through problem solving in mathematical*

Three metacognitive functions are used to describe student behavior such as metacognitive awareness (MA), metacognitive evaluation (ME), and metacognitive regulation (MR). Metacognitive Awareness relates to an individual's awareness of where they are in the learning process, their own knowledge about content knowledge, personal learning strategies, and what has been done and needs to be done. Metacognitive Evaluation refers to the judgment made regarding one's thinking capacities and limitations as they are employed in a particular situation or as a self attributes. Metacognitive Regulation occurs when individuals modify their thinking.

Specifically, for problem number 1: A cube has a surface area of fifty-four square centimeters. What is the volume of the cube?

The formula for the volume V of a cube with edge-length e is: $V = l^3$

To find the volume, Students need the edge-length. Can students use the surface-area information to get what I need? Let's see...

A cube has six sides, each of which is a square; and the edges of the cube's faces are the sides of those squares. The formula for the area of a square with side-length l is $S_s = l^2$. There are six faces so there are six squares, and the cube's total surface area S_T must be: $S_T = 6l^2$. Plugging in the value they gave me, they get:

$54 = 6l^2$ deduce $3 = l$

Since the volume is the cube of the edge-length, and since the units on this cube are centimeters, then: the volume is 27 cubic centimeters

The thinking skills used were defining the problem (Def), representing, establishing criteria (Esc) and setting goals (Seg). For the problem solving 1, the patterns were DefRep, DefEscRep, DefRecRep and DefSegRecRep.

For the problem number 2, We are scheduling a mathematics solve problems by calculator tournament in our school. There are five teams that registered. Each team will play with all the other teams once. How many games will be scheduled?, the thinking skills used by the respondents were defining the problem (Def), representing (Rep), comparing (Co), identifying relations (IR) and verifying (Vr). In problem solving 3 the students had DefRep, DefRepCo, DefIR, and DefRepIRVr.

For the third problem solving: Given $f(x) = ax^2 + bx + c, a \neq 0, \Delta = b^2 - 4ac$ *and* $f(x) \in R[x]$ *We get:*

- $f(x) > 0$ for all x if and only if { $a > 0$ and $\Delta < 0$ }.
- $f(x) \geq 0$ for all x if and only if { $a > 0$ and $\Delta \leq 0$ }.
- $f(x) < 0$ for all x if and only if { $a < 0$ and $\Delta < 0$ }.
- $f(x) \leq 0$ for all x if and only if { $a < 0$ and $\Delta \leq 0$ }.
- $f(x) = 0$ have two solutions x_1, x_2 and real numbers $x_1 < \alpha < x_2$ if and only if $af(\alpha) < 0$.

Example. Suppose finite sequence of real numbers $(a_i), (b_i), (t_i)$ *such that* $0 < a \leq a_i \leq A, 0 < b \leq b_i \leq B$ *and* $t_i \geq 0$ *where for all* $i = 1, \ldots, n$. *We have*

- [Polya]:

$$\frac{1}{4}\left(\sqrt{\frac{ab}{AB}} + \sqrt{\frac{AB}{ab}}\right)^2 \geq \frac{\sum_{i=1}^{n} a_i^2 . \sum_{i=1}^{n} b_i^2}{\left(\sum_{i=1}^{n} a_i b_i\right)^2}.$$

- [Cantorovic]:

$$\frac{(a+A)^2}{4aA}\left(\sum_{i=1}^{n} t_i\right)^2 \geq \left(\sum_{i=1}^{n} t_i a_i\right)\left(\sum_{i=1}^{n} \frac{t_i}{a_i}\right).$$

Proof.

(i) By $f(x) = x^2 - \left(\frac{b}{A} + \frac{B}{a}\right)x + \frac{Bb}{Aa}$ have two solutions $\frac{b}{A}, \frac{B}{a}$.

Since $\frac{b}{A} \leq \frac{b_i}{a_i} \leq \frac{B}{a}$ we deduce the inequality

$$\frac{b_i^2}{a_i^2} - \left(\frac{b}{A} + \frac{B}{a}\right)\frac{b_i}{a_i} + \frac{Bb}{Aa} \leq 0$$

or $b_i^2 - \left(\frac{b}{A} + \frac{B}{a}\right)b_i a_i + \frac{Bb}{Aa} a_i^2 \leq 0$ where $i = 1, \ldots, n$.

Suming up we get:

$$\left(\frac{b}{A} + \frac{B}{a}\right)\sum_{i=1}^{n} b_i a_i \geq \sum_{i=1}^{n} b_i^2 + \frac{Bb}{Aa}\sum_{i=1}^{n} a_i^2 \geq 2\sqrt{\left(\sum_{i=1}^{n} b_i^2\right)\left(\frac{Bb}{Aa}\sum_{i=1}^{n} a_i^2\right)}.$$

Hence

$$\frac{1}{4}\left(\sqrt{\frac{ab}{AB}}+\sqrt{\frac{AB}{ab}}\right)^2=\frac{1}{4}\left(\frac{b}{A}+\frac{B}{a}\right)^2\frac{Aa}{Bb}\geq\frac{\sum_{i=1}^{n}a_i^2.\sum_{i=1}^{n}b_i^2}{(\sum_{i=1}^{n}a_ib_i)^2}.$$

(ii) So $0<a\leq a_i\leq A$ and $t_i\geq 0$ deduce $t_ia_i+\frac{t_iaA}{a_i}\leq(a+A)t_i$ where $i=1,\ldots,n,$ deduce we have $\sum_{i=1}^{n}t_ia_i+\sum_{i=1}^{n}\frac{t_iaA}{a_i}\leq(a+A)\sum_{i=1}^{n}t_i$.

Applying the Cauchy's Inequality, we get $(a+A)(\sum_{i=1}^{n}t_i)\geq 2\sqrt{aA(\sum_{i=1}^{n}t_ia_i)(\sum_{i=1}^{n}\frac{t_i}{a_i})}$ or the Inequality $\frac{(a+A)^2}{4aA}(\sum_{i=1}^{n}t_i)^2\geq(\sum_{i=1}^{n}t_ia_i)(\sum_{i=1}^{n}\frac{t_i}{a_i})$.

The thinking skills utilized by the students for problem number 3, they were recalling (Rec), representing (Rep), identifying relation (IR), and elaborating (Ela).

There were four patterns of thinking processes exhibited by the students relative to this problem solving, namely RecRep, RecRepIR, RecRepEla, and RepEla.

For the fourth problem: In mathematics, a function f defined on a subset of the real numbers with real values is called monotonic if and only if it is either entirely increasing or decreasing. It is called monotonically increasing (also increasing or nondecreasing), if for all x and y such that $x\leq y$ one has $f(x)\leq f(y)$, so f preserves the order. Likewise, a function is called monotonically decreasing (also decreasing or nonincreasing) if, whenever $x\leq y$, then $f(x)\geq f(y)$, so it reverses the order Because the function $f(x)=\frac{(1-x)^2x}{1-2x}$ with the derivative $f'(x)=\frac{(1-x)(4x^2-3x+1)}{(1-2x)^2}>0$ where $x\in(0,1/2)$ thus, $f(x)$ is a continuous monotonically increasing function in $(0,1/2)$. Furthermore $f(0)=0$ and $\lim_{x\to(\frac{1}{2})^-}f(x)=+\infty.$

The continue mobilizing metacognitive skills of the pupils about continuous monotonically increasing function. How should I proceed? Should I move in a different direction?

Set $\alpha=p^2uvt.$ Suppose $g(y)$ is called the inverse function of $f(x)$. We have $g(y)$ is a continuous monotonically increasing function.

Because $f(u)=\frac{(1-u)^2u}{1-2u}=\frac{p^2uvt}{\ell_a^2}$ deduce $u=g(\frac{\alpha}{\ell_a^2}),\ v=g(\frac{\alpha}{\ell_b^2}),\ t=g(\frac{\alpha}{\ell_c^2}).$

From $u+v+t=\frac{a+b+c}{p}=1$ we deduce $g(\frac{\alpha}{\ell_a^2})+g(\frac{\alpha}{\ell_b^2})+g(\frac{\alpha}{\ell_c^2})=1.$

Function $G(\alpha)=g(\frac{\alpha}{\ell_a^2})+g(\frac{\alpha}{\ell_b^2})+g(\frac{\alpha}{\ell_c^2})$ is a continuous monotonically increasing function with $\lim_{\alpha\to 0}G(\alpha)=0,\ \lim_{\alpha\to+\infty}G(\alpha)=\frac{3}{2}.$ We get α_0 such that $g(\frac{\alpha_0}{\ell_a^2})+g(\frac{\alpha_0}{\ell_b^2})+g(\frac{\alpha_0}{\ell_c^2})=1.$

We get specific numbers $u_0=g(\frac{\alpha_0}{\ell_a^2}),$ $v_0=g(\frac{\alpha_0}{\ell_b^2}),\ t_0=g(\frac{\alpha_0}{\ell_c^2})$ và $p_0=\sqrt{\frac{\alpha_0}{u_0v_0t_0}}.$

We deduce $a_0=p_0u_0, b_0=p_0v_0, c_0=p_0t_0.$

Students get the results I expected! Can students apply this way of thinking to other problems or situations? This thinking shows how to "*construct an triangle with compass and straightedge when It begins with the lengths of 3 median lines of triangle*". Is there anything students don't understand any gaps in their knowledge? How might they apply this line of thinking to this new problems.

Suppose we have triangle ABC. We call the intersection of the medians is the centroid, it is G. M is the middle point of BC. We drawing GM ray, set N on GM ray such that $MN=MG.$ We construct an triangle GNC where the lengths of sides $GN=\frac{2}{3}m_a$, $NC=\frac{2}{3}m_b$ and $CG=\frac{2}{3}m_c$,.

Hence, we get triangle ABC.

The students using metacognitive skills, they can prove some this new problems and also present some this problems that seem to be difficult if they are built from traditional discrete thinking. Teacher can encourage students to become more strategic thinkers by helping them focus on the ways they process information. Self-questioning, reflective journal writing, and discussing their thought processes with other students are among the ways that teachers can encourage students to examine and develop their metacognitive processes.

In this fourth problem, The thinking skills of defining the problem (Def), setting goals (Seg),

representing (Rep), verifying (Vr), and identifying relations (IR) were used.
Based on problem solving 4, students had five patterns of thinking process namely DefSegRep, DefSegRepVr, DefSegRepIRVr, DefSegRepIRRep, and DefSegRepIRVrRep.
(North Central Regional Educational Laboratory, 1995), Individuals with well-developed metacognitive skills can think through a problem or approach a learning task, select appropriate strategies, and make decisions about a course of action to resolve the problem or successfully perform the task. They often think about their own thinking processes, taking time to think about and learn from mistakes or inaccuracies.
Summary:
It was noted that the process of solving a problem started with defining of the given problem. Representing skill appeared to be common in most of the patterns of thinking.
There were six patterns of metacognitive actions the students have shown. These were MA-ME, MA-ME-MR, MA-MR, MA-MR-ME, MR-MA-ME, ME-MR.
There was a general tendency among subjects to be metacognitive evaluative in their behavior rather than metacognitive regulative. Students who utilized metacognitive regulation in their solution more often resulted to correct response than those who did not employ this function.
We can be seen, all these metacognitive actions were employed by the students in solving problems. Among the patterns, the generally practiced pattern is MA-ME and followed by MA-ME-MR. The students are sort of metacognitive evaluative (ME) rather than metacognitive regulative (MR). We state that, In the same manner cognitive activity happened before or after metacogntive evaluation or metacognitive regulation took place. Metacognitive experiences usually precede or follow a cognitive activity.
The study gave the insight that subjects who capitalized on metacognitive functions resulted to better quality of answer. Thus it was successful in bringing up the concern that metacognition can be capitalized for better learning. ([Qua16]) state that, teachers can create examples with different levels that will improve students' confidence when tackling open tasks. These results illustrate the importance of metacognitive strategies, which could bring about successful student mathematical problem solving. It could be seen that students could solve math problems successfully; they tried to find various problem solving strategies and could continue solving problems without giving up their efforts to create new problem solving approaches and to express various ways of thinking by using problem solving tools of previously learned ideas and strategies. These findings are in line with Schoenfeld's conclusion ([Sch85]) that a good problem-solver constantly questions their achievement.

4. CONCLUSION

One of the goals of education in Vietnam is to teach students thinking, including metacognitive thinking. There are several ways to promote and develop students' metacognitive thinking. This article focuses on developing students' metacognitive thinking through example in the teaching specific mathematical. The goal of teaching metacognitive strategies is to help students become comfortable with these strategies so that they employ them automatically to learning tasks, focusing their attention, deriving meaning, and making adjustments if something goes wrong. They do not think about these skills while performing them but, if asked what they are doing, they can usually accurately describe their metacognitive processes.

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