ROBUSTNESS OF MODIFIED FACTOR-TYPE ESTIMATORS UNDER NON-RESPONSE MODEL

A. Audu [1], A. A. Adewara [2]

[1] Department of Mathematics, Usmanu Danfodiyo University, P.M.B. 2346, Sokoto, Nigeria
[2] Department of Statistics, University of Ilorin, P.M.B 1515, Ilorin, Kwara State, Nigeria

Corresponding Author: A. Audu, ahmed.audu@udusok.edu.ng

ABSTRACT: In this paper, robustness of suggested modified factor-type estimators have been study under non-response model in relation to other existing related estimators of population mean. Mean square error (MSE) of suggested estimators under non-response model has been obtained and empirical study was done using Data 1, 2, 3 and 4. The robustness of the considered estimators were obtained by averaging their expected MSE and ranked accordingly to their level of efficiency. The results revealed that the efficiency of all the estimators considered increases as the proportion of non-respondent who responded after been re-interviewed increases and the suggested estimators compete favorably with almost the estimators considered in the study.
***KEYWORDS*:** Estimator, Robustness, Mean square error (MSE), Efficiency.

1.0 INTRODUCTION

It is well known especially in human surveys that information is generally not obtained from all the sample units even after callbacks. This is called non-response.
Hansen and Hurwitz ([HH46]) gave a procedure to deal with non-response: Select a sample of size n from the population (denote this sample by s). Let n_1 units respond (sample s_1) and n_2 not respond (sample S_2), for $n_1+n_2=n$. Let out of n_2 , h_2 were re-interviewed and they responded (sample s_{h_2}). The suggested unbiased estimator is given as;

$$\bar{y}_w=\frac{n_1\bar{y}_{s_1}+n_2\bar{y}_{s_{h_2}}}{n} \qquad (1.1)$$

where $\bar{y}_{s_1}=\frac{1}{n_1}\sum_{i\in s_1}y_i$, $\bar{y}_{s_{h_2}}=\frac{1}{h_2}\sum_{i\in s_{h_2}}y_i$

The problem of estimating the parameters such as ratio of two means, population mean and variance when some observations are missing due to random non response has been discussed by Toutenberg and Srivastava ([TS98]), Singh and Joarder ([SJ98]), Singh S. et al. ([SS00]) and Singh and Tracy ([ST01]). The problem of estimation of population mean using information on single auxiliary character has been considered by different authors such as Tabasum and Khan ([TH04, TK06]), Khare and Sinha ([KS07]), Singh and Kumar ([SK11]), Kumar et al. ([K+11]). Also, authors like Singh and Kumar ([SK09]) and Ismail et al. ([IHS15]) have suggested some improved estimators for population mean in the presence of non-response using more information of auxiliary variable like coefficients of variation, skewness, kurtosis etc and from their empirical results, more information of auxiliary variable play significance role in stabilizing the performances of estimators in the presence of non-response.
Definition: Let $A=(d-1)(d-2)$, $B=(d-1)(d-4)$, $C=(d-2)(d-3)(d-4)$,

$$\psi_1=\frac{A+C}{A+fB+C},\ \psi_2=\frac{fB}{A+fB+C},\ \psi_3=\frac{A+fB}{A+fB+C},\ \psi_4=\frac{C}{A+fB+C}\ ,\ P=\psi_3-\psi_1=\psi_2-\psi_4$$

d is an unknown positive real number to be estimated i.e $d\in\Re^+$
The traditional factor-type estimator for population mean was suggested by Singh and Shukla ([SS87, SS93]). This estimator is defined as

$$\bar{y}_{FT} = \bar{y}\left[\frac{(A+C)\bar{X} + fB\bar{x}}{(A+fB)\bar{X} + C\bar{x}}\right] \tag{1.2}$$

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i, \quad \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

$$MSE(\bar{y}_{FT}) = \bar{Y}^2 \frac{1-f}{n}\left(C_y^2 + P^2C_x^2 + 2P\rho C_x C_y\right) \tag{1.3}$$

where $\theta_1 = \frac{fB}{A+fB+C}$, $\theta_2 = \frac{C}{A+fB+C}$, $P = \theta_1 - \theta_2$, $S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^2$,
$S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2$, $C_y = \frac{S_y}{\bar{Y}}$, $C_x = \frac{S_x}{\bar{X}}$
Remark: Since $|fB| < 1$ and $|C| > 1$, it implies that $|P| < 1$.
Shukla ([Shu02]) suggested a factor-type estimator for population mean under two-phase sampling as

$$\bar{y}_{FTd} = \bar{y}_2 \frac{(A+C)\bar{x}_1 + fB\bar{x}_2}{(A+fB)\bar{x}_1 + C\bar{x}_2} \tag{1.4}$$

$$f = \frac{n_2}{N}, \qquad \bar{x} = \frac{1}{n_2}\sum_{i=1}^{n_2} x_i, \; \bar{x}_1 = \frac{1}{n_1}\sum_{i=1}^{n_1} x_i \quad \bar{y} = \frac{1}{n_2}\sum_{i=1}^{n_2} y_i$$

The MSE of $\bar{y}_{FTd}$ under case I and II are respectively

$$MSE(\bar{y}_{FTd})_I = \bar{Y}^2\left[\theta_2 C_y^2 + \theta_3 P^2 C_x^2 + 2\theta_3 P\rho_{xy} C_x C_y\right] \tag{1.5}$$

$$MSE(\bar{y}_{FTd})_{II} = \bar{Y}^2\left[\theta_2 C_y^2 + \theta_4 P^2 C_x^2 + 2\theta_2 P\rho_{xy} C_x C_y\right] \tag{1.6}$$

where $\theta_1 = \frac{1}{n_1} - \frac{1}{N}$, $\theta_2 = \frac{1}{n_2} - \frac{1}{N}$, $\theta_3 = \frac{1}{n_2} - \frac{1}{n_1}$, $\theta_4 = \theta_1 + \theta_2$, $S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^2$,
$S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2$, $S_{xy} = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})(Y_i - \bar{Y})$, $C_y = \frac{S_y}{\bar{Y}}$, $C_x = \frac{S_x}{\bar{X}}$, $\rho_{xy} = \frac{S_{xy}}{S_x S_y}$,
It is observed that the factor-type estimator $\bar{y}_{FTd}$ was more efficient than classical ratio estimator $\bar{y}_r^d$, if $-2C_{yx} < P < 0$ in case I and if $-2C_{yx}(1+\delta)^{-1} < P < 0$ in case II where $\delta = \theta_1\theta_2^{-1}$.
Thakur and Gupta ([TG13]) suggested a linear combination based factor-type estimator $\bar{y}_{FTRP}$ for estimating population mean in sample surveys. The suggested estimator, its bias and MSE are given below;

$$\bar{y}_{FTRP} = f\bar{y}\frac{(A+C)\bar{X} + fB\bar{x}}{(A+fB)\bar{X} + C\bar{x}} + (1-f)\bar{y}\frac{(A+C)\bar{x} + fB\bar{X}}{(A+fB)\bar{x} + C\bar{X}} \tag{1.7}$$

$$MSE(\bar{y}_{FTRP}) = \bar{Y}^2\frac{1-f}{n}\left(C_y^2 + (2f-1)^2P^2C_x^2 + 2(2f-1)P\rho C_x C_y\right) \tag{1.8}$$

Jain and Shukla ([JS13]) suggested factor-type estimators $\bar{y}_{FT1}$ and $\bar{y}_{FT2}$ in multiprocessor environment for estimation of ready queue processing time. The suggested estimators, their biases and MSEs are given below;

$$\bar{y}_{FT1} = \bar{y}\frac{9\bar{X} + 2f\bar{x}}{(6+2f)\bar{X} + 3\bar{x}} \tag{1.9}$$

$$MSE(\bar{y}_{FT1}) = \bar{Y}^2\frac{1-f}{n}\left(C_y^2 + \frac{4f^2-12f+9}{81+36f+4f^2}C_x^2 + \frac{4f-6}{18+4f}\rho C_x C_y\right) \tag{1.10}$$

$$\bar{y}_{FT2} = \bar{y}\frac{22\bar{X} + 5f\bar{x}}{(10+5f)\bar{X} + 12\bar{x}} \tag{1.11}$$

$$MSE(\bar{y}_{FT2}) = \bar{Y}^2\frac{1-f}{n}\left(C_y^2 + \frac{25f^2-120f+144}{484+220f+25f^2}C_x^2 + \frac{10f-24}{44+10f}\rho C_x C_y\right) \tag{1.12}$$

Shukla et. al. ([SJV13]) suggested a transformed factor-type estimator for population mean in multiprocessor environment for estimation of ready queue processing time. The suggested estimator is given below as;

$$\bar{y}_{TFT} = \bar{y}\,\frac{(A+C)(d-1)\bar{X} + fB(d\bar{X}-\bar{x})}{(A+fB)(d-1)\bar{X} + C(d\bar{X}-\bar{x})} \tag{1.13}$$

The MSE of the suggested estimator are respectively given as follows;

$$MSE(\bar{y}_{FT}) = \bar{Y}^2\frac{1-f}{n}\left(C_y^2 + \left(\frac{P}{d-1}\right)^2 C_x^2 + 2\frac{P}{d-1}\rho C_x C_y\right), \quad for\ d > 1 \tag{1.14}$$

2.0 METHODOLOGY

Having studied the above estimators, we suggested the following four factor-type estimators. The suggested estimator denoted by $\bar{y}_{FTAA}$, $\bar{y}_{FTAA}^{(d)}$, $\bar{y}_{FTAA}^{\beta_1(d)}$, $\bar{y}_{FTAA}^{\beta_2(d)}$ and their properties are given as;

$$\bar{y}_{FTAA} = \bar{y}\,\frac{\left[(A+C)\bar{X} + fB\bar{x}\right]a_x + \left[A+C+fB\right]b_x}{\left[(A+fB)\bar{X} + C\bar{x}\right]a_x + \left[A+C+fB\right]b_x} \tag{2.1}$$

$$MSE(\bar{y}_{FTAA}) = \bar{Y}^2\frac{1-f}{n}\left[C_y^2 + \delta_x^2 P^2 C_x^2 + 2\delta_x P\rho_{xy}C_xC_y\right] \tag{2.2}$$

$$MSE(\bar{y}_{FTAA})_{\min} = \bar{Y}^2\frac{1-f}{n}C_y^2\left[1-\rho_{xy}^2\right] \tag{2.3}$$

$$\bar{y}_{FTAA}^{(d)} = \bar{y}_2\,\frac{\left[(A+C)\bar{x}_1 + fB\bar{x}_2\right]a_x + \left[A+C+fB\right]b_x}{\left[(A+fB)\bar{x}_1 + C\bar{x}_2\right]a_x + \left[A+C+fB\right]b_x} \tag{2.4}$$

$$MSE\left(\bar{y}_{FTAA}^{(d)}\right)_I = \bar{Y}^2\left[\theta_2C_y^2 + \theta_3\delta_x^2P^2C_x^2 + 2\theta_3\delta_xP\rho_{xy}C_xC_y\right] \tag{2.5}$$

$$MSE\left(\bar{y}_{FTAA}^{(d)}\right)_{II} = \bar{Y}^2\left[\theta_2C_y^2 + \theta_4\delta_x^2P^2C_x^2 + 2\theta_2\delta_xP\rho_{xy}C_xC_y\right] \tag{2.6}$$

$$MSE\left(\bar{y}_{FTAA}^{(d)}\right)_{I\min} = \bar{Y}^2C_y^2\left[\theta_2 - \theta_3\rho_{xy}^2\right] \tag{2.7}$$

$$MSE\left(\bar{y}_{FTAA}^{(d)}\right)_{II\min} = \bar{Y}^2C_y^2\left[\theta_2 - \frac{\theta_2^2}{\theta_4}\rho_{xy}^2\right] \tag{2.8}$$

$$\bar{y}_{FTAA}^{\beta_1(d)} = \bar{y}_2\left[\frac{(A+C)\bar{x}_1 + fB\bar{x}_2}{(A+fB)\bar{x}_1 + C\bar{x}_2}\right]\left[\frac{a_z\bar{Z}+b_z}{a_z\bar{z}_1+b_z}\right]^{\beta_1} \tag{2.9}$$

$$MSE\left(\bar{y}_{FTAA}^{\beta_1(d)}\right)_I = \bar{Y}^2\left[\theta_2C_y^2 + \theta_3C_x^2P\left(P+2C_{yx}\right) + \theta_1\beta_1\delta_zC_z^2\left(\beta_1\delta_z - 2C_{yz}\right)\right] \tag{2.10}$$

$$MSE\left(\bar{y}_{FTAA}^{\beta_1(d)}\right)_{II} = \bar{Y}^2\left[\theta_2C_y^2 + C_x^2P\left\{\theta_2\left(P+2C_{yx}\right) + \theta_3P\right\} + \theta_3\beta_1\delta_zC_z^2\left(\beta_1\delta_z + 2PC_{xz}\right)\right] \tag{2.11}$$

$$MSE_{\min}\left(\bar{y}_{FTAA}^{\beta_1(d)}\right)_I = \bar{Y}^2C_y^2\left[\theta_2 - \theta_3\rho_{xy}^2 - \theta_1\rho_{yz}^2\right] \tag{2.12}$$

$$MSE_{\min}\left(\bar{y}_{FTAA}^{\beta_1(d)}\right)_{II} = \bar{Y}^2\theta_2C_y^2\left[1-\rho_{xy}^2/\left(1+(\theta_1/\theta_2)-(\theta_1/\theta_2)\rho_{xz}^2\right)\right] \tag{2.13}$$

$$MSE\left(\bar{y}_{FTAA}^{\beta_2(d)}\right)_I = \bar{Y}^2\left[\theta_2C_y^2 + \theta_3C_x^2P\left(P-2C_{yx}\right) + \theta_3\beta_2\delta_zC_z^2\left(\beta_1\delta_z - 2C_{yz} - 2PC_{xz}\right)\right] \tag{2.14}$$

$$\begin{aligned} MSE\left(\bar{y}_{FTAA}^{\beta_2(d)}\right)_{II} = \bar{Y}^2\Big[&\theta_2C_y^2 + \theta_2C_x^2P\left(P+2C_{yx}\right) + \theta_3\beta_2\delta_zC_z^2\left(\beta_2\delta_z - 2PC_{xz}\right) \\ &\theta_2\delta_z\beta_2C_z^2\left(\delta_z\beta_2 - 2C_{yz} - 2PC_{xz}\right) + \theta_3P^2C_x^2\Big] \end{aligned} \tag{2.15}$$

$$MSE_{\min}\left(\bar{y}_{FTAA}^{\beta_2(d)}\right)_I = \bar{Y}^2C_y^2\left[\theta_2 - \theta_3\left(\rho_{xy}^2 + \rho_{yz}^2 + 2\rho_{xy}\rho_{xz}\rho_{yz}\right)/\left(1-\rho_{xz}^2\right)\right] \tag{2.16}$$

$$MSE_{\min}\left(\bar{y}_{FTAA}^{\beta_2(d)}\right)_{II} = \bar{Y}^2\theta_2\left[C_y^2 + \frac{\left(\rho_{yz}\rho_{xz}-\rho_{xy}C_y\right)}{\theta_4^2\left(1-\rho_{xz}^2\right)^2}\left\{\rho_{yz}\rho_{xz}\theta_2\theta_4\left(1-2C_y\right)+\theta_4\rho_{xy}C_y\left(2-\theta_2\right)\right.\right.$$
$$\left.2\theta_4\rho_{xz}^2\rho_{xy}C_y\left(\theta_2-1\right)\right\}+\frac{\left(C_{yz}-C_{yx}C_{xz}\right)}{\theta_4^2\left(1-\rho_{xz}^2\right)^2}\left\{\theta_2\rho_{yz}C_yC_z\left(\theta_4-2\right)\right. \tag{2.17}$$
$$\left.\left.+\theta_2\rho_{xz}C_yC_z\left(2\rho_{yz}\rho_{xz}-\theta_4\rho_{xy}\right)\right\}\right]$$

3.0 MSE OF THE SUGGESTED ESTIMATORS UNDER NON-RESPONSE MODEL

If $\bar{y}$ is replaced by $\bar{y}^*$ in the proposed estimators then the proposed can be written under non-response model as;

$$\bar{y}_{FTAA}^{*} = \left(\frac{n_{S1}\bar{y}_{s1}+n_{s2}\bar{y}_{sh2}}{n}\right)\left(\frac{\left(\left(A+C\right)\bar{X}+fB\bar{x}\right)a_x+\left(A+C+fB\right)b_x}{\left(\left(A+fB\right)\bar{X}+C\bar{x}\right)a_x+\left(A+C+fB\right)b_x}\right) \tag{3.1}$$

$$\bar{y}_{FTAA}^{(d)*} = \left(\frac{n_{S1}\bar{y}_{s1}+n_{s2}\bar{y}_{sh2}}{n_2}\right)\left(\frac{\left(\left(A+C\right)\bar{x}_1+fB\bar{x}_2\right)a_x+\left(A+C+fB\right)b_x}{\left(\left(A+fB\right)\bar{x}_1+C\bar{x}_2\right)a_x+\left(A+C+fB\right)b_x}\right) \tag{3.2}$$

$$\bar{y}_{FTAA}^{\beta_1(d)*} = \left(\frac{n_{S1}\bar{y}_{s1}+n_{s2}\bar{y}_{sh2}}{n_2}\right)\left(\frac{\left(A+C\right)\bar{x}_1+fB\bar{x}_2}{\left(A+fB\right)\bar{x}_1+C\bar{x}_2}\right)\left(\frac{a_z\bar{Z}+b_z}{a_z\bar{z}_1+b_z}\right)^{\beta_1} \tag{3.3}$$

$$\bar{y}_{FTAA}^{\beta_2(d)*} = \left(\frac{n_{S1}\bar{y}_{s1}+n_{s2}\bar{y}_{sh2}}{n_2}\right)\left(\frac{\left(A+C\right)\bar{x}_1+fB\bar{x}_2}{\left(A+fB\right)\bar{x}_1+C\bar{x}_2}\right)\left(\frac{a_z\bar{z}_1+b_z}{a_z\bar{z}_2+b_z}\right)^{\beta_2} \tag{3.4}$$

In order to get the MSEs of $\bar{y}_{FTAA}^{*}$, $\bar{y}_{FTAA}^{(d)*}$, $\bar{y}_{FTAA}^{\beta_1(d)*}$ and $\bar{y}_{FTAA}^{\beta_2(d)*}$, we defined an error term $\Delta_{\bar{y}}^{*}=\left(\bar{y}^*-\bar{Y}\right)/\bar{Y}$ such that $\left|\Delta_{\bar{y}}^{*}\right|<1$ and $E\left(\Delta_{\bar{y}}^{*}\right)=0$, $E\left(\Delta_{\bar{y}}^{*2}\right)=\frac{1}{\bar{Y}^2}\left(\left(\frac{1}{n}-\frac{1}{N}\right)S_y^2+\frac{W_2\left(k-1\right)}{n}S_{y(s2)}^2\right)$, $E\left(\Delta_{\bar{x}}\Delta_{\bar{y}}^{*}\right)=E\left(\Delta_{\bar{y}}^{*}\Delta_{\bar{z}}\right)=0$, then

$$MSE\left(\bar{y}_{FTAA}^{*}\right)=MSE\left(\bar{y}_{FTAA}\right)+\frac{W_2\left(k-1\right)}{n\bar{Y}^2}S_{y(s2)}^2 \tag{3.5}$$

$$MSE\left(\bar{y}_{FTAA}^{(d)*}\right)=MSE\left(\bar{y}_{FTAA}^{(d)}\right)+\frac{W_2\left(k-1\right)}{n_2\bar{Y}^2}S_{y(s2)}^2 \tag{3.6}$$

$$MSE\left(\bar{y}_{FTAA}^{\beta_1(d)*}\right)=MSE\left(\bar{y}_{FTAA}^{\beta_1(d)}\right)+\frac{W_2\left(k-1\right)}{n_2\bar{Y}^2}S_{y(s2)}^2 \tag{3.7}$$

$$MSE\left(\bar{y}_{FTAA}^{\beta_2(d)*}\right)=MSE\left(\bar{y}_{FTAA}^{\beta_2(d)}\right)+\frac{W_2\left(k-1\right)}{n_2\bar{Y}^2}S_{y(s2)}^2 \tag{3.8}$$

where $W_2=\frac{N}{N_{s2}}$ and $k=\frac{n_{s2}}{n_{h2}}$.

4.0 EMPIRICAL STUDY

In this section, efficiency of these suggested factor-type estimators over some existing related estimators were investigated empirically.

4.1. Data 1: Khare and Rehman ([KR15])

Y: Number of Agricultural labor, X: Area of the Village (hectares), Z: Number of cultivators in the village.

$$\bar{Y} = 137.9271,\ \bar{X} = 144.8720,\ \bar{Z} = 185.188,\ S_y = 182.5012, C_x = 0.8115$$

$$S_{y(2)} = 287.4204,\ \rho_{xy} = 0.773,\ \rho_{yz} = 0.786,\ \rho_{xz} = 0.819,\ W_2 = 0.25$$

$$N = 96,\ n_2 = 24,\ n_1 = 60$$

Table 1: MSE of $\bar{y}_{FTAA}, \bar{y}_{FTAA}^{(d)}, \bar{y}_{FTAA}^{\beta_1(d)}$ and $\bar{y}_{FTAA}^{\beta_2(d)}$ under Non-Response Model using Data 1

Estimators	$(1/k)$				Ave. MSE	Rank
	$(1/5)$	$(1/4)$	$(1/3)$	$(1/2)$		
Sample mean	985.32	985.32	985.32	985.32	985.32	20th
Cochran ([Coc42])	426.94	425.94	424.94	423.94	425.44	11th
Srivenkataramana ([Sri80])	9039.9	9038.9	9037.9	9036.9	9038.4	26th
Sisodia and Diwivedi ([SD81])	426.9	425.94	424.9	423.9	425.41	8th
Singh and Tailor ([ST03])	426.9	425.9	424.9	423.93	425.40	7th
Singh et. al. ([SUC04])	9039.9	9038.9	9037.9	9036.9	9038.4	27th
Upadhyaya and Singh ([US99])	426.9	425.94	424.9	423.9	425.41	9th
Upadhyaya and Singh ([US99])	426.9	425.94	424.9	423.9	425.41	9th
Sukhatme ([Suk62])	546	545	544	543	544.5	14th
Srivenkataramana ([Sri80]) –two-phase	2789.1	2788.1	2787.1	2786.1	2787.6	24th
Choudhury and Singh ([CS12])	256.1	255.1	254.1	253.1	254.6	1st
Chand ([Cha75])	5494.6	5493.6	5492.6	5491.6	5493.1	25th
Singh and Upadhayaya ([SU01])	727.2	726.2	725.2	724.2	725.7	18th
Singh et al. ([SCS07])	397.3	396.3	395.3	394.3	395.8	3rd
Singh and Upadhayaya ([SU01])	727.2	726.2	725.2	724.2	725.7	18th
Singh ([Sin01])	417.4	416.4	415.4	414.4	415.9	5th
Malik and Tailor ([MT13])	545.7	544.8	543.9	542.8	544.3	13th
Singh and Shukla ([SS87, SS93])	990.3	989.3	988.3	987.3	988.8	21st
Shukla ([Shu02])	990.3	989.3	988.4	987.3	988.83	23rd
Jain and Shukla ([JS13]) 1	629.3	628.3	627.3	626.3	627.8	17th
Jain and Shukla ([JS13]) 2	482.3	481.3	480.3	479.4	480.83	12th
Thakur and Gupta ([TG13])	990.3	989.3	988.3	987.3	988.8	22nd
Shukla et. al. ([SJV13])	590.3	589.9	588.3	587.9	589.1	15th
Suggested $\bar{y}_{FTAA}$	396.7	396.6	396.5	396.4	396.55	4th
Suggested $\bar{y}_{FTAA}^{(d)}$	614.28	614.24	614.2	614.1	614.21	16th
Suggested $\bar{y}_{FTAA}^{\beta_1(d)}$	392.5	392.47	392.43	392.39	392.45	2nd
Suggested $\bar{y}_{FTAA}^{\beta_2(d)}$	423.39	423.44	423.48	423.52	423.46	6th

Table 1 shows the biases and MSEs of the suggested estimators and those of some existing related estimators under Non-response model. Estimation of biases and MSE under the model was done using Data C1. The robustness of the considered estimators were obtained by averaging their expected MSEs and ranked accordingly to their level of efficiency. The results revealed that out of the twenty-seven competing estimators, $\bar{y}_{FTAA}^{\beta_1(d)}$, $\bar{y}_{FTAA}$, $\bar{y}_{FTAA}^{\beta_2(d)}$ and $\bar{y}_{FTAA}^{(d)}$ are ranked second, fourth, sixth and sixteenth respectively.

4.2. Data 2: Singh and Kumar ([SK11])

Y: Weight (kg) of the children, X: Skull circumference (cm) of the children, Z: Chest circumference (cm) of the children

$$\bar{Y} = 19.4968,\ \bar{X} = 51.1726,\ \bar{Z} = 55.1726,\ C_y = 0.15613, C_x = 0.03006$$

$$C_{y(2)} = 287.4204,\ \rho_{xy} = 0.328,\ \rho_{yz} = 0.846,\ \rho_{xz} = 0.297,\ W_2 = 0.25$$

$$N = 95,\ n_2 = 24,\ n_1 = 35$$

Table 2: MSE of $\bar{y}_{FTAA}, \bar{y}_{FTAA}^{(d)}, \bar{y}_{FTAA}^{\beta_1(d)}$ and $\bar{y}_{FTAA}^{\beta_2(d)}$ under Non-Response Model using Data 2

Estimators	$(1/k)$				Average	Rank
	$(1/5)$	$(1/4)$	$(1/3)$	$(1/2)$		
Sample mean	0.2886	0.2886	0.2886	0.2886	0.2886	5th
Cochran ([Coc42])	25.905	23.641	21.378	19.114	22.5095	22nd
Srivenkataramana ([Sri80])	292.3	290.03	287.77	285.5	288.9	26th
Sisodia and Diwivedi ([SD81])	25.905	23.641	21.378	19.113	22.50925	21st
Singh and Tailor ([ST03])	25.905	23.641	21.378	19.114	22.5095	23rd
Singh et. al. ([SUC04])	292.3	290.03	287.77	285.5	288.9	27th
Upadhyaya and Singh ([US99])	25.905	23.641	21.378	19.114	22.5095	24th
Upadhyaya and Singh ([US99])	25.905	23.641	21.378	19.114	22.5095	24th
Sukhatme ([Suk62])	16.308	14.044	11.781	9.5168	12.91245	19th
Srivenkataramana ([Sri80]) –two-phase	17.568	15.304	13.04	10.776	14.172	20th
Choudhury and Singh ([CS12])	10.331	8.0669	5.8032	3.5393	6.9351	14th
Chand ([Cha75])	9.78	7.5162	5.2524	2.9886	6.3843	13th
Singh and Upadhayaya ([SU01])	9.3768	7.113	4.8492	2.5854	5.9811	11th
Singh et al. ([SCS07])	9.211	6.9472	4.6834	2.4196	5.8153	6th
Singh and Upadhayaya ([SU01])	9.3768	7.113	4.8492	2.5854	5.9811	11th
Singh ([Sin01])	16.189	13.925	11.661	9.3971	12.79303	17th
Malik and Tailor ([MT13])	16.303	14.039	11.775	9.5114	12.9071	18th
Singh and Shukla ([SS87, SS93])	9.3437	7.0799	4.8161	2.5523	5.948	7th
Shukla ([Shu02])	9.3437	7.0799	4.8161	2.5523	5.948	7th
Jain and Shukla ([JS13]) 1	10.645	8.3811	6.1173	3.8535	7.249225	15th
Jain and Shukla ([JS13]) 2	13.473	11.209	8.9455	6.6817	10.0773	16th
Thakur and Gupta ([TG13])	9.3437	7.0799	4.8161	2.5523	5.948	7th
Shukla et. al. ([SJV13])	9.3437	7.0799	4.8161	2.5523	5.948	10th
Suggested $\bar{y}_{FTAA}$	0.2581	0.258	0.2578	0.2577	0.2579	4th
Suggested $\bar{y}_{FTAA}^{(d)}$	0.2312	0.231	0.2309	0.2307	0.23095	3rd
Suggested $\bar{y}_{FTAA}^{\beta_1(d)}$	0.1564	0.1563	0.1561	0.156	0.1562	1st
Suggested $\bar{y}_{FTAA}^{\beta_2(d)}$	0.1577	0.1575	0.1574	0.1572	0.15745	2nd

Table 2 shows the biases and MSEs of the suggested estimators and those of some existing related estimators under Non-response model. Computations of biases and MSEs under the model were done using Data 2. The robustness of the considered estimators were obtained by averaging their expected MSEs and ranked accordingly to their level of efficiency. The results revealed that out of the twenty-seven competing estimators, $\bar{y}_{FTAA}^{\beta_1(d)}$, $\bar{y}_{FTAA}^{\beta_2(d)}$, $\bar{y}_{FTAA}^{(d)}$ and $\bar{y}_{FTAA}$ are ranked first, second, third and fourth respectively.

4.3. Dataset 3: Sanaullah et al. ([S+15])

Y: Food expenditure, X: Household earning, Z: Total expenditure in month of May

$\bar{Y} = 47.9805,\ \bar{X} = 18746.55,\ \bar{Z} = 19124.75,\ S_y = 21.4256, S_x = 16625.33$

$S_{y(2)} = 20.4752,\ \rho_{xy} = -0.4777,\ \rho_{yz} = -0.4422,\ \rho_{xz} = 0.9138,\ W_2 = 0.10$

$N = 6940,\ n_2 = 750,\ n_1 = 1874$

Table 3: MSE of $\bar{y}_{FTAA}, \bar{y}_{FTAA}^{(d)}, \bar{y}_{FTAA}^{\beta_1(d)}$ and $\bar{y}_{FTAA}^{\beta_2(d)}$ under Non-Response Model using Data 3

Estimators	$(1/k)$				Average	Rank
	$(1/5)$	$(1/4)$	$(1/3)$	$(1/2)$		
Sample mean	0.546	0.546	0.546	0.546	0.546	9th
Product	1.6634	1.6634	1.6634	1.6634	1.6634	16th
Pandey and Dubey ([PD88]) t_8	1.6634	1.6634	1.6634	1.6634	1.6634	16th
Singh and Tailor ([ST03]) t_9	1.6634	1.6634	1.6634	1.6634	1.6634	16th
Singh ([Sin03]) t_{10}	1.6634	1.6634	1.6634	1.6634	1.6634	16th
Singh et. al. ([SUC04]) t_{11}	1.6634	1.6634	1.6634	1.6634	1.6634	16th
Upadhyaya and Singh ([US99]) t_{12}	1.6634	1.6634	1.6634	1.6634	1.6634	16th
Singh ([Sin03]) t_{13}	1.6634	1.6634	1.6634	1.6634	1.6634	16th
Singh ([Sin03]) t_{14}	1.6634	1.6634	1.6634	1.6634	1.6634	16th
Upadhyaya and Singh ([US99]) t_{15}	1.6634	1.6634	1.6634	1.6634	1.6634	16th
Singh ([Sin03]) t_{16}	1.6634	1.6634	1.6634	1.6634	1.6634	16th
Choudhury and Singh ([CS12])	0.2552	0.2552	0.2552	0.2552	0.2552	1st
Chand ([Cha75])	0.2825	0.2825	0.2824	0.2824	0.28245	2nd
Singh and Upadhayaya ([SU01])	0.8178	0.8178	0.8177	0.8177	0.81775	12th
Singh et al. ([SCS07])	0.4273	0.4273	0.4272	0.4272	0.42725	4th
Singh and Upadhayaya ([SU01])	0.8178	0.8178	0.8177	0.8177	0.81775	12th
Singh ([Sin01])	2.6556	2.6555	2.6554	2.6555	2.6555	27th
Malik and Tailor ([MT13])	2.6907	2.6906	2.6906	2.6906	2.690625	28th
Singh and Shukla ([SS87, SS93])	0.5462	0.546	0.5459	0.5458	0.545975	7th
Shukla ([Shu02])	0.5462	0.546	0.5459	0.5458	0.545975	7th
Jain and Shukla ([JS13]) 1	1.0814	1.0813	1.08132	1.0813	1.08133	14th
Jain and Shukla ([JS13]) 2	1.6716	1.6716	1.6716	1.6716	1.6716	26th
Thakur and Gupta ([TG13])	0.546	0.546	0.546	0.546	0.546	9th
Shukla et. al. ([SJV13])	0.546	0.546	0.546	0.546	0.546	9th
Suggested $\bar{y}_{FTAA}$	0.4215	0.4214	0.4214	0.4214	0.421425	3rd
Suggested $\bar{y}_{FTAA}^{(d)}$	0.4623	0.4622	0.4622	0.4621	0.4622	6th
Suggested $\bar{y}_{FTAA}^{\beta_1(d)}$	0.4273	0.4273	0.4272	0.4272	0.42725	5th
Suggested $\bar{y}_{FTAA}^{\beta_2(d)}$	1.256	1.256	1.2561	1.2561	1.25605	15th

Table 3 shows the biases and MSEs of the suggested estimators and those of some existing related estimators under Non-response model. Computations of MSE under the model were done using Data 3. The robustness of the considered estimators were obtained by averaging their expected MSEs and ranked accordingly to their level of efficiency. The results revealed that out of the twenty-seven competing estimators, $\bar{y}_{FTAA}$, $\bar{y}_{FTAA}^{\beta_1(d)}$, $\bar{y}_{FTAA}^{(d)}$ and $\bar{y}_{FTAA}^{\beta_2(d)}$ are ranked third, fifth, sixth and fifteenth respectively.

4.4. Dataset 4: Sanaullah et al. ([S+15])
Y: Food expenditure, X: House earn , Z: Total expenditure in month of May

$$\bar{Y} = 48.0556,\ \bar{X} = 14303.98,\ \bar{Z} = 14742.47,\ S_y = 22.1319, S_x = 12861.4$$

$$S_{y(2)} = 21.7407,\ \rho_{xy} = -0.4406,\ \rho_{yz} = -0.3547,\ \rho_{xz} = 0.8035,\ W_2 = 0.10$$

$$N = 1678,\ n_2 = 181,\ n_1 = 453$$

Table 4: MSE of $\bar{y}_{FTAA}, \bar{y}_{FTAA}^{(d)}, \bar{y}_{FTAA}^{\beta_1(d)}$ and $\bar{y}_{FTAA}^{\beta_2(d)}$ under Non-Response Model using Data C4

Estimators	$(1/k)$				Average MSE	Rank
	$(1/5)$	$(1/4)$	$(1/3)$	$(1/2)$		
Sample mean	2.4154	2.4151	2.41485	2.4146	2.414993	8th
Product	7.4642	7.4640	7.4637	7.4634	7.463825	17th
Pandey and Dubey ([PD88]) t_8	7.4642	7.4640	7.4637	7.4634	7.463825	17th
Singh and Tailor ([ST03]) t_9	7.4643	7.4640	7.46372	7.4633	7.46383	17th
Singh ([Sin03]) t_{10}	7.4643	7.4640	7.46372	7.4633	7.46383	17th
Singh et. al. ([SUC04]) t_{11}	7.4643	7.4640	7.46372	7.4633	7.46383	17th
Upadhyaya and Singh ([US99]) t_{12}	7.4643	7.4640	7.46372	7.4633	7.46383	17th
Singh ([Sin03]) t_{13}	7.4643	7.4640	7.46372	7.4633	7.46383	17th
Singh ([Sin03]) t_{14}	7.4643	7.4640	7.4637	7.4633	7.463825	17th
Upadhyaya and Singh ([US99]) t_{15}	7.4643	7.4640	7.4637	7.4633	7.463825	17th
Singh ([Sin03]) t_{16}	7.4643	7.4640	7.4637	7.4633	7.463825	17th
Choudhury and Singh ([CS12])	1.1089	1.1085	1.10819	1.1079	1.108373	2nd
Chand ([Cha75])	1.4976	1.4973	1.4970	1.4967	1.49715	3rd
Singh and Upadhayaya ([SU01])	3.6034	3.6032	3.6029	3.6026	3.603025	13th
Singh et al. ([SCS07])	2.0007	2.0004	2.0001	1.9998	2.00025	5th
Singh and Upadhayaya ([SU01])	3.6034	3.6031	3.6029	3.6026	3.603	13th
Singh ([Sin01])	11.305	11.305	11.304	11.303	11.30425	27th
Malik and Tailor ([MT13])	11.405	11.405	11.404	11.403	11.40425	28th
Singh and Shukla ([SS87, SS93])	2.4154	2.4151	2.4149	2.4146	2.415	8th
Shukla ([Shu02])	2.4154	2.4151	2.4149	2.4146	2.415	8th
Jain and Shukla ([JS13]) 1	4.6177	4.6174	4.6172	4.6169	4.6173	15th
Jain and Shukla ([JS13]) 2	7.0834	7.0831	7.0828	7.0826	7.082975	16th
Thakur and Gupta ([TG13])	2.4154	2.4151	2.4149	2.4146	2.415	8th
Shukla et. al. ([SJV13])	2.4154	2.4151	2.4149	2.4146	2.415	8th
Suggested $\bar{y}_{FTAA}$	1.9467	1.9465	1.9462	1.9459	1.946325	4th
Suggested $\bar{y}_{FTAA}^{(d)}$	2.1000	2.0997	2.0994	2.0991	2.09955	7th
Suggested $\bar{y}_{FTAA}^{\beta_1(d)}$	2.0007	2.0004	2.0001	1.9998	2.00025	5th
Suggested $\bar{y}_{FTAA}^{\beta_2(d)}$	0.2039	0.2036	0.2034	0.2031	0.2035	1st

Table 4 shows the MSEs, average of MSEs and ranks of suggested estimators and those of some existing related estimators under Non-response model. The results revealed that out of the twenty-seven competing estimators, $\bar{y}_{FTAA}^{\beta_2(d)}$, $\bar{y}_{FTAA}$, $\bar{y}_{FTAA}^{\beta_1(d)}$ and $\bar{y}_{FTAA}^{(d)}$ are ranked first, second, fourth, fifth and seventh respectively.

4.0 RESULTS AND CONCLUSIONS

Robustness of the considered estimators and some existing related estimators under non-response model were investigated using four real life data. The results revealed that the suggested estimators display high level of robustness under non-response model as they compete favourably with almost existing related estimators considered.

REFERENCES

[Cha75] **L. Chand** - *Some ratio type estimator based on two or more auxiliary variables*. Ph.D.dissertation. Lowa State University. Ames, Lowa (unpublished), 1975.

[Coc42] **W. G. Cochran** - *Sampling Theory when the Sampling Units are of Unequal Sizes*. Journ. American Statistical Association. 37: 199-212, 1942.

[CS12] **S. Choudhury, B. K. Singh** – *A class of chain ratio-product type estimators with two auxiliary variables under double sampling scheme*. Jour. Of the Korean Stat. Soc. 41, 247-256, 2012.

[HH46] **M. H. Hansen, W. N. Hurwitz** - *The problem of non-response in sample surveys*. Jour. Amer. Statist. Assoc., 41, 517- 529, 1946.

[IHS15] **M. Ismail, M. Hanif, M. Q. Shahbaz** - *Generalized Estimators for Population Mean in the Presence of Non-Response for Two-Phase Sampling*. Pak. J. Statist., 31 (3), 295-306, 2015.

[JS13] **A. Jain, D. Shukla** - *Estimation of Ready Queue Processing Time using Factor-Type (F-T) Estimator in Multiprocessor Environment*. International Journal of Advanced Computer technology, 2 (8): 256-260, 2013.

[KS07] **B. B. Khare, R. R. Sinha** - *Estimation of the Ratio of the two Population Means using Multi-auxiliary characters in the presence of Non-response*. Statistical Techniques in Life Testing, Reliability, Sampling Theory and Quality Control, 163- 171, 2007.

[KR15] **B. B. Khare, H. U. Rehman** - *Improved Ratio in Regression type Estimator for Population Mean using Coefficient of Variation of the Study Character in the Presence of Non-Response*. International Journal of Technology Innovations and Research, 14, 1-7, 2015.

[K+11] **S. Kumar, H. P. Singh, S. Bhougal, R. Gupta** - *Under double sampling a general ratio type estimator in presence of non-response.* Journal of Mathematics and Statistics, 40(4), 589-599, 2011.

[MT13] **K. A. Malik, R. Tailor** - *Ratio Type Estimator of Population Mean in Double Sampling*. International Journal of Advanced Mathematics and Statistics. 1 (1): 34-39, 2013.

[PD88] **B. N. Pandey, V. Dubey** - *Modified product estimator using coefficient of variation of auxiliary variate*. Assam Statistical Rev., 2(2), 64-66, 1988.

[Shu02] **D. Shukla** - *F-T estimator under two phase sampling*, Metron, 59 (1-2): 253-263, 2002.

[Sin01] **G. N. Singh** - *On the use of transformed auxiliary variable in the estimation of population mean in two phase sampling*. Statistics in Transition. 5(3), 405-416, 2001.

[Sin03] **G. N. Singh** - *On the improvement of product method of estimation in sample surveys*. Jour. Ind. Soc. Agric. Statistics. 56(3): 267-275, 2003.

[Sri80] **T. Srivenkatarama** - *A dual to ratio estimator in sample survey.* Biometrica, 67(1), 199-204, 1980.

[Suk62] **B. Y. Sukhatme** - *Some ratio type estimators in two-phase sampling*. J. Amer. Statist. Assoc., 57: 628-632, 1962.

[SD81] **B. V. S. Sisodaiya, V. K. Dwivedi** - *A modified ratio estimator using coefficient of variation of auxillary variable*. Jour. Ind. Soc. Agr. Statistics. 33: 13-18, 1981.

[SJ98] **S. Singh, A. Joarder** - *Estimation of finite population variance using random Non-response in survey sampling*. Metrika, 241-249, 1998.

[SK09] **H. P. Singh, S. Kumar** - *A General Procedure of Estimating the Population Mean in the Presence of Non-response under Double Sampling using Auxiliary Information*. SORT, 33, 71-84, 2009.

[SK11] **H. P. Singh, S. Kumar** - *Combination of ratio and regression estimators in presence of non-response*. Brazilian journal of Probability and Statistics, 25(2), 205-217, 2011.

[SS87] **V. K. Singh, D. Shukla** - *One parameter family of factor type ratio estimators.* Metron International Journal of Statistics, 45 (12): 273-283, 1987.

[SS93] **V. K. Singh, D. Shukla** - *An efficient one-parameter family of factor-type estimator in sample surveys*. Metron International Journal of Statistics, 55: 139-159, 1993.

[ST01] **H. P. Singh, D. S. Tracy** - *Estimation of population mean in presence of non-random non-response in sample surveys*. Statistica, LXI, 2, 231-248, 2001.

[ST03] **H. P. Singh, R. Tailor** - *Use of known correlation coefficient in estimating the finite Population mean.* Statistics in Transition. 64: 555-560, 2003.

[SU01] **G. N. Singh, L. N. Upadhyaya** - *An Empirical Study of Modified Ratio Estimators in Two Phase Sampling in Presence of Coefficient of Variation of the Auxiliary Variable*, Statistics in Transition, 5(2): 319-326, 2001.

[SCS07] **R. Singh, P. Chauhan, N. Sawan** - *A Family of Estimators for Estimating Population Mean Using Known Correlation Coefficient in Two Phase Sampling*. Statistics in Transition, 8(1), 89-96, 2007.

[SJV13] **S. Shukla, A. Jain, K. Verma** - *Estimation of Ready Queue Processing Time using Transformed Factor-Type (T-F-T) Estimator in Multiprocessor Environment*. International Journal of Computer Applications, 79 (16): 40-48, 2013.

[SSM00] **S. Singh, R. Singh, N. S. Mangat** - *Some alternative strategies to Moors' model in randomized response sampling.* J. Statist. Planning Infer., 83, 243-255. 2000.

[SUC04] **H. P. Singh, L. N. Upadhyaya, P. Chandra** - *A general family of estimators for estimating population mean using two auxiliary variables in two-phase sampling*. Statistics in Transition, 6(7): 1055-1077, 2004.

[S+15] **A. Sanaullah, M. Noo-ul-amin, M. Hanif, R. Singh** - *Generalized exponential chain estimators using two auxiliary variables for stratified sampling with non-response*. Sci. Int. (Lohore), 27 (2), 901-915, 2015.

[TG13] **N. S. Thakur, K. Gupta** - *A Study of Linear Combination Based Factor-Type Estimator for Estimating Population Mean in Sample Survey*, Journal of Reliability and Statistical Studies. 6(2): 81-90, 2013.

[TK04] **R. Tabasum, I. A. Khan** - *Double sampling for ratio estimation with non-response,* Jour. Ind. Soc. Agril. Statist., 58(3), 300-306, 2004.

[TK06] **R. Tabasum, I. A. Khan** - *Double Sampling Ratio Estimator for the Population Mean in Presence of Non-Response*. Assam Statist. Review, 20(1), 73- 83, 2006.

[TS98] **H. Toutenberg, V. K. Srivastava** - *Estimation of ratio of population means in survey sampling when some observations are missing*. Metrika, (48)177-187, 1998.

[US99] **L. N. Upadhyaya, H. P. Singh** - *Use of transformed auxiliary variable in estimating the finite population mean*. Biometrical Journal, 41(5), 627-636, 1999.