EFFICIENCY OF SOME MODIFIED RATIO TYPE ESTIMATORS USING PRODUCT OF SAMPLE SIZE AND PARAMETERS OF AUXILIARY VARIABLE FOR ESTIMATING POPULATION MEAN

S. A. Suleiman [1], A. A. Adewara [2]

[1] Department of Statistics, Kwara State Polytechnic, Ilorin, Nigeria
[2] Department of Statistics, University of Ilorin, Ilorin, Nigeria

Corresponding Author: S. A. Suleiman, sikirusuleiman097@gmail.com

ABSTRACT: In this paper, we modified twenty-eight ratio type estimators for estimation of population mean of the study variable earlier suggested by Gupta and Yadav ([GY17]) using product of sample size and population parameter of auxiliary variable. The expressions for the bias and mean square errors of the newly modified ratio type estimators have been obtained up to the first order of approximation. A comparison has been made with the mentioned existing ratio estimators of population mean using the same data set used by Gupta and Yadav ([GY17]) for easy justification. The results obtained on the Mean Square Errors shows that the newly modified ratio type estimators perform better vis-à-vis the earlier suggested Gupta and Yadav ([GY17]) existing ratio type estimators but the newly modified ratio type Estimator, t^*_{27}, perform better, hence, recommended for usage in Sampling.
KEYWORDS: Ratio Estimator, Auxiliary information, Sample size, Bias, Mean Squared Error, Efficiency.

1. INTRODUCTION

In sample survey, the main objective is to obtain the estimators of parameters of interest with increase precision. It is common practice to use the auxiliary variable for improving the precision of the estimate of a parameter. Use of such auxiliary information is made through the ratio method of estimation to obtain an improved estimator of population mean when is highly positively correlated with variable under study. In this paper, we have modified about twenty-eight Ratio Type Estimators earlier suggested by Gupta and Yadav ([GY17]) for improved estimation of population mean with higher efficiencies.

Let the population under consideration consists of N distinct and identifiable units and let (x_i,y_i), i=1,2,…,n be a two variable sample of size n taken from bivariate variable (X,Y) through simple random sampling without sampling scheme. Let $\bar{X}$ and $\bar{Y}$ be the population means of the auxiliary and the study variable respectively, and let $\bar{x}$ and $\bar{y}$ be respective sample means and both unbiased estimators of $\bar{X}$ and $\bar{Y}$ respectively. Let the correlation coefficient between the variables X and Y be denoted by ρ ([Coc40]).

2. EXISTING ESTIMATORS UNDER REVIEW

Let the sample mean $\bar{y}$ by define as:

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \quad (1)$$

Its variance up to the first order of approximation be:

$$v(\bar{y}) = \frac{1-f}{n} s_y^2 \quad (2)$$

The usual ratio estimator of population be denoted as:

$$t_x = \bar{y}\frac{\bar{X}}{\bar{x}} \quad (3)$$

Its bias and mean squared error, up to the first order of approximation respectively be:

$$B(t_R) = \frac{1-f}{n}\frac{1}{\bar{X}}\left[R_1 S_x^2 - \rho S_y S_x\right] \quad (4)$$

$$MSE(t_R) = \frac{1-f}{n}\left[S_y^2 + R_1^2 S_x^2 - 2R_1\rho S_y S_x\right] \quad (5)$$

Where, $R_1 = \frac{\bar{Y}}{\bar{X}}$ ([Coc40, Coc77]).

Some of the earlier suggested Gupta and Yadav ([GY17]) existing Ratio Type Estimators are as shown in Table 1.

3. NEWLY MODIFIED RATIO TYPE ESTIMATORS

Motivated by the work of Gupta and Yadav ([GY17]), we have used a product of sample size and parameters of auxiliary variable given as t_i^*, $i=1,2,...,28$ as also shown in Table 1, where $\tau=\rho/\beta_1$ was used for t^*_{26}, t^*_{27} and t^*_{28} ,

$\bar{y} = \bar{Y}(1+e_0), \bar{x} = \bar{X}(1+e_1), E(e_i) = 0, (i = 0,1), E(e_0^2) = \frac{1-f}{n}C_y^2$

$E(e_1^2) = \frac{1-f}{n}C_x^2, E(e_0 e_1) = \frac{1-f}{n}\rho C_y C_x, f = \frac{n}{N}, C_y^2 = \frac{S_y^2}{\bar{Y}^2}, C_x^2 = \frac{S_x^2}{\bar{X}^2}$

$$B(t_j^*) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_j^{*2}, (j = 2, \dots, 29), \quad (6)$$

$$MSE(t_j^*) = \frac{1-f}{n}\left[R_j^{*2}S_x^2 + S_y^2(1-\rho^2)\right], (j = 2, \dots, 29) \quad (7)$$

and $R_j^{*2}, (j = 2, \dots, 29)$

Table 1: Expressions for the MSE of the Twenty - Eight Ratio Type Estimators being considered

Estimator	Constant	Bias	MSE
$t_2 = \frac{y + b(\bar{X}-\bar{x})}{(\bar{x}+C_x)}(\bar{X}+C_x)$ **Kadilar and Cingi ([KC04])**	$R_2 = \frac{\bar{Y}}{\bar{X}+C_x}$	$B(t_2) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_2^2$	$MSE(t_2) = \frac{1-f}{n}\left[R_2^2 S_x^2 + S_y^2(1-\rho^2)\right]$
$t_2^* = \frac{y + b(\bar{X}-\bar{x})}{(\bar{x}+C_x n)}(\bar{X}+C_x n)$ **modified estimator t₂**	$R_2^* = \frac{\bar{Y}}{\bar{X}+C_x n}$	$B(t_2^*) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_2^{*2}$	$MSE(t_2^*) = \frac{1-f}{n}\left[R_2^{*2} S_x^2 + S_y^2(1-\rho^2)\right]$
$t_3 = \frac{y + b(\bar{X}-\bar{x})}{(\bar{x}+\beta_2)}(\bar{X}+\beta_2)$ **Kadilar and Cingi ([KC04])**	$R_3 = \frac{\bar{Y}}{\bar{X}+\beta_2}$	$B(t_3) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_3^2$	$MSE(t_3) = \frac{1-f}{n}\left[R_3^2 S_x^2 + S_y^2(1-\rho^2)\right]$
$t_3^* = \frac{y + b(\bar{X}-\bar{x})}{(\bar{x}+\beta_2 n)}(\bar{X}+\beta_2 n)$ **modified estimator t₃**	$R_3^* = \frac{\bar{Y}}{\bar{X}+\beta_2 n}$	$B(t_3^*) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_3^{*2}$	$MSE(t_3^*) = \frac{1-f}{n}\left[R_3^{*2} S_x^2 + S_y^2(1-\rho^2)\right]$
$t_4 = \frac{y + b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+C_x)}(\bar{X}\beta_2+C_x)$ **Kadilar and Cingi ([KC04])**	$R_4 = \frac{\bar{Y}\beta_2}{\bar{X}\beta_2+C_x}$	$B(t_4) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_4^2$	$MSE(t_4) = \frac{1-f}{n}\left[R_4^2 S_x^2 + S_y^2(1-\rho^2)\right]$
$t_4^* = \frac{y + b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+C_x n)}(\bar{X}\beta_2+C_x n)$ **modified estimator t₄**	$R_4^* = \frac{\bar{Y}\beta_2}{\bar{X}\beta_2+C_x n}$	$B(t_4^*) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_4^{*2}$	$MSE(t_4^*) = \frac{1-f}{n}\left[R_4^{*2} S_x^2 + S_y^2(1-\rho^2)\right]$
$t_5 = \frac{y + b(\bar{X}-\bar{x})}{(\bar{x}C_x+\beta_2)}(\bar{X}C_x+\beta_2)$ **Kadilar and Cingi ([KC04])**	$R_5 = \frac{\bar{Y}C_x}{\bar{X}C_x+\beta_2}$	$B(t_5) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_5^2$	$MSE(t_5) = \frac{1-f}{n}\left[R_5^2 S_x^2 + S_y^2(1-\rho^2)\right]$
$t_5^* = \frac{y + b(\bar{X}-\bar{x})}{(\bar{x}C_x+\beta_2 n)}(\bar{X}C_x+\beta_2 n)$ **modified estimator t₅**	$R_5^* = \frac{\bar{Y}C_x}{\bar{X}C_x+\beta_2 n}$	$B(t_5^*) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_5^{*2}$	$MSE(t_5^*) = \frac{1-f}{n}\left[R_5^{*2} S_x^2 + S_y^2(1-\rho^2)\right]$
$t_6 = \frac{y + b(\bar{X}-\bar{x})}{(\bar{x}+\rho)}(\bar{X}+\rho)$ **Kadilar and Cingi ([KC04])**	$R_6 = \frac{\bar{Y}}{\bar{X}+\rho}$	$B(t_6) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_6^2$	$MSE(t_6) = \frac{1-f}{n}\left[R_6^2 S_x^2 + S_y^2(1-\rho^2)\right]$
$t_6^* = \frac{y + b(\bar{X}-\bar{x})}{(\bar{x}+\rho n)}(\bar{X}+\rho n)$ **modified estimator t₆**	$R_6^* = \frac{\bar{Y}}{\bar{X}+\rho n}$	$B(t_6^*) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_6^{*2}$	$MSE(t_6^*) = \frac{1-f}{n}\left[R_6^{*2} S_x^2 + S_y^2(1-\rho^2)\right]$
$t_7 = \frac{y + b(\bar{X}-\bar{x})}{(\bar{x}C_x+\rho)}(\bar{X}C_x+\rho)$ **Kadilar and Cingi ([KC06])**	$R_7 = \frac{\bar{Y}C_x}{\bar{X}C_x+\rho}$	$B(t_7) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_7^2$	$MSE(t_7) = \frac{1-f}{n}\left[R_7^2 S_x^2 + S_y^2(1-\rho^2)\right]$
$t_7^* = \frac{y + b(\bar{X}-\bar{x})}{(\bar{x}C_x+\rho n)}(\bar{X}C_x+\rho n)$ **modified estimator t₇**	$R_7^* = \frac{\bar{Y}C_x}{\bar{X}C_x+\rho n}$	$B(t_7^*) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_7^{*2}$	$MSE(t_7^*) = \frac{1-f}{n}\left[R_7^{*2} S_x^2 + S_y^2(1-\rho^2)\right]$

Estimator	Constant	Bias	MSE
$t_8 = \frac{y + b(\bar{X} - \bar{x})}{(\bar{x}\rho + C_x)}(\bar{X}\rho + C_x)$ **Kadilar and Cingi ([KC06])** $t_8^* = \frac{y + b(\bar{X} - \bar{x})}{(\bar{x}\rho + C_x n)}(\bar{X}\rho + C_x n)$ **modified estimator t8**	$R_8 = \frac{\bar{Y}}{\bar{X}\rho + C_x}$ $R_8^* = \frac{\bar{Y}}{\bar{X}\rho + C_x n}$	$B(t_8) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_8^2$ $B(t_8^*) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_8^{*2}$	$MSE(t_8) = \frac{1-f}{n}[R_8^2 S_x^2 + S_y^2(1-\rho^2)]$ $MSE(t_8^*) = \frac{1-f}{n}[R_8^{*2} S_x^2 + S_y^2(1-\rho^2)]$
$t_9 = \frac{y + b(\bar{X} - \bar{x})}{(\bar{x}\beta_2 + \rho)}(\bar{X}\beta_2 + \rho)$ **Kadilar and Cingi ([KC06])** $t_9^* = \frac{y + b(\bar{X} - \bar{x})}{(\bar{x}\beta_2 + \rho n)}(\bar{X}\beta_2 + \rho n)$ **modified estimator t9**	$R_9 = \frac{\bar{Y}\beta_2}{\bar{X}\beta_2 + \rho}$ $R_9^* = \frac{\bar{Y}\beta_2}{\bar{X}\beta_2 + \rho n}$	$B(t_9) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_9^2$ $B(t_9^*) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_9^{*2}$	$MSE(t_9) = \frac{1-f}{n}[R_9^2 S_x^2 + S_y^2(1-\rho^2)]$ $MSE(t_9^*) = \frac{1-f}{n}[R_9^{*2} S_x^2 + S_y^2(1-\rho^2)]$
$t_{10} = \frac{y + b(\bar{X} - \bar{x})}{(\bar{x}\rho + \beta_2)}(\bar{X}\rho + \beta_2)$ **Kadilar and Cingi ([KC06])** $t_{10}^* = \frac{y + b(\bar{X} - \bar{x})}{(\bar{x}\,\rho + \beta_2 n)}(\bar{X}\rho + \beta_2 n)$ **modified estimator t10**	$R_{10} = \frac{\bar{Y}\rho}{\bar{X}\rho + \beta_2}$ $R_{10}^* = \frac{\bar{Y}\rho}{\bar{X}\rho + \beta_2 n}$	$B(t_{10}) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{10}^2$ $B(t_{10}^*) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{10}^{*2}$	$MSE(t_{10}) = \frac{1-f}{n}[R_{10}^2 S_x^2 + S_y^2(1-\rho^2)]$ $MSE(t_{10}^*) = \frac{1-f}{n}[R_{10}^{*2} S_x^2 + S_y^2(1-\rho^2)]$
$t_{11} = \frac{y + b(\bar{X} - \bar{x})}{(\bar{x} + \beta_1)}(\bar{X} + \beta_1)$ **Yan and Tian ([YT10])** $t_{11}^* = \frac{y + b(\bar{X} - \bar{x})}{(\bar{x} + \beta_1 n)}(\bar{X} + \beta_1 n)$ **modified estimator t11**	$R_{11} = \frac{\bar{Y}}{\bar{X} + \beta_1}$ $R_{11}^* = \frac{\bar{Y}}{\bar{X} + \beta_1 n}$	$B(t_{11}) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{11}^2$ $B(t_{11}^*) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{11}^{*2}$	$MSE(t_{11}) = \frac{1-f}{n}[R_{11}^2 S_x^2 + S_y^2(1-\rho^2)]$ $MSE(t_{11}^*) = \frac{1-f}{n}[R_{11}^{*2} S_x^2 + S_y^2(1-\rho^2)]$
$t_{12} = \frac{y + b(\bar{X} - \bar{x})}{(\bar{x}\beta_1 + \beta_2)}(\bar{X}\beta_1 + \beta_2)$ **Yan and Tian ([YT10])** $t_{12}^* = \frac{y + b(\bar{X} - \bar{x})}{(\bar{x}\,\beta_1 + \beta_2 n)}(\bar{X}\beta_1 + \beta_2 n)$ **modified estimator t12**	$R_{12} = \frac{\bar{Y}\beta_1}{\bar{X}\beta_1 + \beta_2}$ $R_{12}^* = \frac{\bar{Y}\beta_1}{\bar{X}\beta_1 + \beta_2 n}$	$B(t_{12}) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{12}^2$ $B(t_{12}^*) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{12}^{*2}$	$MSE(t_{12}) = \frac{1-f}{n}[R_{12}^2 S_x^2 + S_y^2(1-\rho^2)]$ $MSE(t_{12}^*) = \frac{1-f}{n}[R_{12}^{*2} S_x^2 + S_y^2(1-\rho^2)]$
$t_{13} = \frac{y + b(\bar{X} - \bar{x})}{(\bar{x} + M_d)}(\bar{X} + M_d)$ **Subramani and Kumarpandiyan ([SJ12])** $t_{13}^* = \frac{y + b(\bar{X} - \bar{x})}{(\bar{x} + M_d n)}(\bar{X} + M_d n)$ **modified estimator t13**	$R_{13} = \frac{\bar{Y}}{\bar{X} + M_d}$ $R_{13}^* = \frac{\bar{Y}}{\bar{X} + M_d n}$	$B(t_{13}) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{13}^2$ $B(t_{13}^*) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{13}^{*2}$	$MSE(t_{13}) = \frac{1-f}{n}[R_{13}^2 S_x^2 + S_y^2(1-\rho^2)]$ $MSE(t_{13}^*) = \frac{1-f}{n}[R_{13}^{*2} S_x^2 + S_y^2(1-\rho^2)]$
$t_{14} = \frac{y + b(\bar{X} - \bar{x})}{(\bar{x}C_x + M_d)}(\bar{X}C_x + M_d)$ **Subramani and Kumarpandiyan ([SJ12])** $t_{14}^* = \frac{y + b(\bar{X} - \bar{x})}{(\bar{x}C_x + M_d n)}(\bar{X}C_x + M_d n)$ **modified estimator t14**	$R_{14} = \frac{\bar{Y}C_x}{\bar{X}C_x + M_d}$ $R_{14}^* = \frac{\bar{Y}C_x}{\bar{X}C_x + M_d n}$	$B(t_{14}) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{14}^2$ $B(t_{14}^*) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{14}^{*2}$	$MSE(t_{14}) = \frac{1-f}{n}[R_{14}^2 S_x^2 + S_y^2(1-\rho^2)]$ $MSE(t_{14}^*) = \frac{1-f}{n}[R_{14}^{*2} S_x^2 + S_y^2(1-\rho^2)]$

Estimator	Constant	Bias	MSE
$t_{15} = \frac{y + b(\bar{X} - \bar{x})}{(\bar{x}\beta_1 + M_d)}(\bar{X}\beta_1 + M_d)$ **Subramani and Kumarpandiyan ([SJ12])** $t_{15}^* = \frac{y + b(\bar{X} - \bar{x})}{(\bar{x}\beta_1 + M_d n)}(\bar{X}\beta_1 + M_d n)$ **modified estimator t15**	$R_{15} = \frac{\bar{Y}\beta_1}{\bar{X}\beta_1 + M_d}$ $R_{15}^* = \frac{\bar{Y}\beta_1}{\bar{X}\beta_1 + M_d n}$	$B(t_{15}) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{15}^2$ $B(t_{15}^*) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{15}^{*2}$	$MSE(t_{15}) = \frac{1-f}{n}[R_{15}^2 S_x^2 + S_y^2(1-\rho^2)]$ $MSE(t_{15}^*) = \frac{1-f}{n}[R_{15}^{*2} S_x^2 + S_y^2(1-\rho^2)]$
$t_{16} = \frac{y + b(\bar{X} - \bar{x})}{(\bar{x}\beta_2 + M_d)}(\bar{X}\beta_2 + M_d)$ **Subramani and Kumarpandiyan ([SJ12])** $t_{16}^* = \frac{y + b(\bar{X} - \bar{x})}{(\bar{x}\beta_2 + M_d n)}(\bar{X}\beta_2 + M_d n)$ **modified estimator t16**	$R_{16} = \frac{\bar{Y}\beta_2}{\bar{X}\beta_2 + M_d}$ $R_{16}^* = \frac{\bar{Y}\beta_2}{\bar{X}\beta_2 + M_d n}$	$B(t_{16}) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{16}^2$ $B(t_{16}^*) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{16}^{*2}$	$MSE(t_{16}) = \frac{1-f}{n}[R_{16}^2 S_x^2 + S_y^2(1-\rho^2)]$ $MSE(t_{16}^*) = \frac{1-f}{n}[R_{16}^{*2} S_x^2 + S_y^2(1-\rho^2)]$
$t_{17} = \frac{y + b(\bar{X} - \bar{x})}{(\bar{x} + QD)}(\bar{X} + QD)$ **Jeelani *et al* ([JMM13])** $t_{17}^* = \frac{y + b(\bar{X} - \bar{x})}{(\bar{x} + QDn)}(\bar{X} + QDn)$ **modified estimator t17**	$R_{17} = \frac{\bar{Y}}{\bar{X} + QD}$ $R_{17}^* = \frac{\bar{Y}}{\bar{X} + QDn}$	$B(t_{17}) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{17}^2$ $B(t_{17}^*) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{17}^{*2}$	$MSE(t_{17}) = \frac{1-f}{n}[R_{17}^2 S_x^2 + S_y^2(1-\rho^2)]$ $MSE(t_{17}^*) = \frac{1-f}{n}[R_{17}^{*2} S_x^2 + S_y^2(1-\rho^2)]$
$t_{18} = \frac{y + b(\bar{X} - \bar{x})}{(\bar{x} + G)}(\bar{X} + G)$ **Abid *et al* ([A+16])** $t_{18}^* = \frac{y + b(\bar{X} - \bar{x})}{(\bar{x} + Gn)}(\bar{X} + Gn)$ **modified estimator t18**	$R_{18} = \frac{\bar{Y}}{\bar{X} + G}$ $R_{18}^* = \frac{\bar{Y}}{\bar{X} + Gn}$	$B(t_{18}) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{18}^2$ $B(t_{18}^*) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{18}^{*2}$	$MSE(t_{18}) = \frac{1-f}{n}[R_{18}^2 S_x^2 + S_y^2(1-\rho^2)]$ $MSE(t_{18}^*) = \frac{1-f}{n}[R_{18}^{*2} S_x^2 + S_y^2(1-\rho^2)]$
$t_{19} = \frac{y + b(\bar{X} - \bar{x})}{(\bar{x}\rho + G)}(\bar{X}\rho + G)$ **Abid *et al* ([A+16])** $t_{19}^* = \frac{y + b(\bar{X} - \bar{x})}{(\bar{x}\rho + Gn)}(\bar{X}\rho + Gn)$ **modified estimator t19**	$R_{19} = \frac{\bar{Y}\rho}{\bar{X}\rho + G}$ $R_{19}^* = \frac{\bar{Y}\rho}{\bar{X}\rho + Gn}$	$B(t_{19}) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{19}^2$ $B(t_{19}^*) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{19}^{*2}$	$MSE(t_{19}) = \frac{1-f}{n}[R_{19}^2 S_x^2 + S_y^2(1-\rho^2)]$ $MSE(t_{19}^*) = \frac{1-f}{n}[R_{19}^{*2} S_x^2 + S_y^2(1-\rho^2)]$
$t_{20} = \frac{y + b(\bar{X} - \bar{x})}{(\bar{x}C_x + G)}(\bar{X}C_x + G)$ **Abid *et al* ([A+16])** $t_{20}^* = \frac{y + b(\bar{X} - \bar{x})}{(\bar{x}C_x + Gn)}(\bar{X}C_x + Gn)$ **modified estimator t20**	$R_{20} = \frac{\bar{Y}C_x}{\bar{X}C_x + G}$ $R_{20}^* = \frac{\bar{Y}C_x}{\bar{X}C_x + Gn}$	$B(t_{20}) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{20}^2$ $B(t_{20}^*) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{20}^{*2}$	$MSE(t_{20}) = \frac{1-f}{n}[R_{20}^2 S_x^2 + S_y^2(1-\rho^2)]$ $MSE(t_{20}^*) = \frac{1-f}{n}[R_{20}^{*2} S_x^2 + S_y^2(1-\rho^2)]$
$t_{21} = \frac{y + b(\bar{X} - \bar{x})}{(\bar{x} + D)}(\bar{X} + D)$ **Abid *et al* ([A+16])** $t_{21}^* = \frac{y + b(\bar{X} - \bar{x})}{(\bar{x} + Dn)}(\bar{X} + Dn)$ **modified estimator t21**	$R_{21} = \frac{\bar{Y}}{\bar{X} + D}$ $R_{21}^* = \frac{\bar{Y}}{\bar{X} + Dn}$	$B(t_{21}) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{21}^2$ $B(t_{21}^*) = \frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{21}^{*2}$	$MSE(t_{21}) = \frac{1-f}{n}[R_{21}^2 S_x^2 + S_y^2(1-\rho^2)]$ $MSE(t_{21}^*) = \frac{1-f}{n}[R_{21}^{*2} S_x^2 + S_y^2(1-\rho^2)]$

Estimator	Constant	Bias	MSE
$t_{22}=\frac{y+b(\bar X-\bar x)}{(\bar x\rho+D)}(\bar X\rho+D)$ **Abid *et al* ([A+16])** $t_{22}^{*}=\frac{y+b(\bar X-\bar x)}{(\bar x\rho+Dn)}(\bar X\rho+Dn)$ **modified estimator t22**	$R_{22}=\frac{\bar Y\rho}{\bar X\rho+D}$ $R_{22}^{*}=\frac{\bar Y\rho}{\bar X\rho+Dn}$	$B(t_{22})=\frac{1-f}{n}\frac{S_x^2}{\bar Y}R_{22}^2$ $B(t_{22}^{*})=\frac{1-f}{n}\frac{S_x^2}{\bar Y}R_{22}^{*2}$	$MSE(t_{22})=\frac{1-f}{n}[R_{22}^2S_x^2+S_y^2(1-\rho^2)]$ $MSE(t_{22}^{*})=\frac{1-f}{n}[R_{22}^{*2}S_x^2+S_y^2(1-\rho^2)]$
$t_{23}=\frac{y+b(\bar X-\bar x)}{(\bar xC_x+D)}(\bar XC_x+D)$ **Abid *et al* ([A+16])** $t_{23}^{*}=\frac{y+b(\bar X-\bar x)}{(\bar xC_x+Dn)}(\bar XC_x+Dn)$ **modified estimator t23**	$R_{23}=\frac{\bar YC_x}{\bar XC_x+D}$ $R_{23}^{*}=\frac{\bar YC_x}{\bar XC_x+Dn}$	$B(t_{23})=\frac{1-f}{n}\frac{S_x^2}{\bar Y}R_{23}^2$ $B(t_{23}^{*})=\frac{1-f}{n}\frac{S_x^2}{\bar Y}R_{23}^{*2}$	$MSE(t_{23})=\frac{1-f}{n}[R_{23}^2S_x^2+S_y^2(1-\rho^2)]$ $MSE(t_{23}^{*})=\frac{1-f}{n}[R_{23}^{*2}S_x^2+S_y^2(1-\rho^2)]$
$t_{24}=\frac{y+b(\bar X-\bar x)}{(\bar x+S_{pw})}(\bar X+S_{pw})$ **Abid *et al* ([A+16])** $t_{24}^{*}=\frac{y+b(\bar X-\bar x)}{(\bar x+S_{pw}n)}(\bar X+S_{pw}n)$ **modified estimator t24**	$R_{24}=\frac{\bar Y}{\bar X+S_{pw}}$ $R_{24}^{*}=\frac{\bar Y}{\bar X+S_{pw}n}$	$B(t_{24})=\frac{1-f}{n}\frac{S_x^2}{\bar Y}R_{24}^2$ $B(t_{24}^{*})=\frac{1-f}{n}\frac{S_x^2}{\bar Y}R_{24}^{*2}$	$MSE(t_{24})=\frac{1-f}{n}[R_{24}^2S_x^2+S_y^2(1-\rho^2)]$ $MSE(t_{24}^{*})=\frac{1-f}{n}[R_{24}^{*2}S_x^2+S_y^2(1-\rho^2)]$
$t_{25}=\frac{y+b(\bar X-\bar x)}{(\bar x\rho+S_{pw})}(\bar X\rho+S_{pw})$ **Abid *et al* ([A+16])** $t_{25}^{*}=\frac{y+b(\bar X-\bar x)}{(\bar x\rho+S_{pw}n)}(\bar X\rho+S_{pw}n)$ **modified estimator t25**	$R_{25}=\frac{\bar Y\rho}{\bar X\rho+S_{pw}}$ $R_{25}^{*}=\frac{\bar Y\rho}{\bar X\rho+S_{pw}n}$	$B(t_{25})=\frac{1-f}{n}\frac{S_x^2}{\bar Y}R_{25}^2$ $B(t_{25}^{*})=\frac{1-f}{n}\frac{S_x^2}{\bar Y}R_{25}^{*2}$	$MSE(t_{25})=\frac{1-f}{n}[R_{25}^2S_x^2+S_y^2(1-\rho^2)]$ $MSE(t_{25}^{*})=\frac{1-f}{n}[R_{25}^{*2}S_x^2+S_y^2(1-\rho^2)]$
$t_{26}=\frac{y+b(\bar X-\bar x)}{(\bar xC_x+S_{pw})}(\bar XC_x+S_{pw})$ **Abid *et al* ([A+16])** $t_{26}^{*}=\frac{y+b(\bar X-\bar x)}{(\bar xC_x+S_{pw}n)}(\bar XC_x+S_{pw}n)$ **modified estimator t26**	$R_{26}=\frac{\bar YC_x}{\bar XC_x+S_{pw}}$ $R_{26}^{*}=\frac{\bar YC_x}{\bar XC_x+S_{pw}n}$	$B(t_{26})=\frac{1-f}{n}\frac{S_x^2}{\bar Y}R_{26}^2$ $B(t_{26}^{*})=\frac{1-f}{n}\frac{S_x^2}{\bar Y}R_{26}^{*2}$	$MSE(t_{26})=\frac{1-f}{n}[R_{26}^2S_x^2+S_y^2(1-\rho^2)]$ $MSE(t_{26}^{*})=\frac{1-f}{n}[R_{26}^{*2}S_x^2+S_y^2(1-\rho^2)]$
$t_{27}=\frac{y+b(\bar X-\bar x)}{(\tau\bar x+G)}(\tau\bar X+G)$ **Gupta and Yadav ([GY17])** $t_{27}^{*}=\frac{y+b(\bar X-\bar x)}{(\tau\bar x+Gn)}(\tau\bar X+Gn)$ **modified estimator t27**	$R_{27}=\frac{\bar Y\tau}{\tau\bar X+G}$ $R_{27}^{*}=\frac{\bar Y\tau}{\tau\bar X+Gn}$	$B(t_{27})=\frac{1-f}{n}\frac{S_x^2}{\bar Y}R_{27}^2$ $B(t_{27}^{*})=\frac{1-f}{n}\frac{S_x^2}{\bar Y}R_{27}^{*2}$	$MSE(t_{27})=\frac{1-f}{n}[R_{27}^2S_x^2+S_y^2(1-\rho^2)]$ $MSE(t_{27}^{*})=\frac{1-f}{n}[R_{27}^{*2}S_x^2+S_y^2(1-\rho^2)]$
$t_{28}=\frac{y+b(\bar X-\bar x)}{(\tau\bar x+D)}(\tau\bar X+D)$ **Gupta and Yadav ([GY17])** $t_{28}^{*}=\frac{y+b(\bar X-\bar x)}{(\tau\bar x+Dn)}(\tau\bar X+Dn)$ **modified estimator t28**	$R_{28}=\frac{\bar Y\tau}{\tau\bar X+D}$ $R_{28}^{*}=\frac{\bar Y\tau}{\tau\bar X+Dn}$	$B(t_{28})=\frac{1-f}{n}\frac{S_x^2}{\bar Y}R_{28}^2$ $B(t_{28}^{*})=\frac{1-f}{n}\frac{S_x^2}{\bar Y}R_{28}^{*2}$	$MSE(t_{28})=\frac{1-f}{n}[R_{28}^2S_x^2+S_y^2(1-\rho^2)]$ $MSE(t_{28}^{*})=\frac{1-f}{n}[R_{28}^{*2}S_x^2+S_y^2(1-\rho^2)]$

Estimator	Constant	Bias	MSE
$t_{29}=\frac{y+b(\bar{X}-\bar{x})}{(\tau\bar{x}+S_{pw})}(\tau\bar{X}+S_{pw})$	$R_{29}=\frac{\bar{Y}\tau}{\tau\bar{X}+S_{pw}}$	$B(t_{29})=\frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{29}^2$	$MSE(t_{29})=\frac{1-f}{n}[R_{29}^2S_x^2+S_y^2(1-\rho^2)]$
Gupta and Yadav ([GY17])			
$t_{29}^*=\frac{y+b(\bar{X}-\bar{x})}{(\tau\bar{x}+S_{pw}n)}(\tau\bar{X}+S_{pw}n)$	$R_{29}^*=\frac{\bar{Y}\tau}{\tau\bar{X}+S_{pw}n}$	$B(t_{29}^*)=\frac{1-f}{n}\frac{S_x^2}{\bar{Y}}R_{29}^{*2}$	$MSE(t_{29}^*)=\frac{1-f}{n}[R_{29}^{*2}S_x^2+S_y^2(1-\rho^2)]$
modified estimator t_{29}			

4. EFFICIENCY COMPARISON

(i) Estimator t_j^* is better than sample mean, $\bar{y}$ whenever:

$$MSE(t_j^*)-V(\bar{y})\le 0$$
or,
$$[R_j^{*2}S_x^2-\rho^2S_y^2]\le 0$$
or,
$$R_j^{*2}\le\frac{\rho^2S_y^2}{S_x^2}$$
or,
$$R_j^{*2}\le\pm\frac{S_y}{S_x},(j=2,3,\dots,29) \qquad (8)$$

(ii) Estimator t_j^* is better than sample mean, t_r ([Coc40]) whenever:

$$MSE(t_j^*)-MSE(t_r)\le 0$$
or,
$$[(R_j^{*2}-R_1^2)S_x^2-\rho^2S_y^2+2R_1\rho S_xS_y]\le 0$$
or,
$$(R_j^{*2}-R_1^2)S_x^2\le\rho^2S_y^2-2R_1\rho S_xS_y,(j=2,3,\dots,29) \qquad (9)$$

(iii) Estimator t_j^* is better than sample mean, t_j ([KC04, KC06]) whenever:

$$MSE(t_j^*)-MSE(t_j)\le 0$$
or,
$$[R_j^{*2}S_x^2-R_j^2S_x^2]\le 0$$
or,
$$R_j^*\le\pm R_j,(j=2,3,\dots,10) \qquad (10)$$

(iv) Estimator t_j^* is better than sample mean, t_j ([YT10]) whenever:

$$MSE(t_j^*)-MSE(t_j)\le 0$$
or,
$$[R_j^{*2}S_x^2-R_j^2S_x^2]\le 0$$
or,
$$R_j^*\le\pm R_j,(j=11,12) \qquad (11)$$

(v) Estimator t_j^* is better than sample mean, t_j ([SJ12]) whenever:

$$MSE(t_j^*)-MSE(t_j)\le 0$$
or,
$$[R_j^{*2}S_x^2-R_j^2S_x^2]\le 0$$
or,
$$R_j^*\le\pm R_j,(j=13,14,15,16) \qquad (12)$$

(vi) Estimator t_j^* is better than sample mean, t_j ([JMM13]) whenever:
$$MSE(t_j^*)-MSE(t_j)\le 0$$
or,
$$[R_j^{*2}S_x^2-R_j^2S_x^2]\le 0$$
or,
$$R_j^*\le\pm R_j,(j=17) \qquad (13)$$

(vii) Estimator t_j^* is better than sample mean, t_j ([A+16]) whenever:

$$MSE(t_j^*)-MSE(t_j)\le 0$$
or,
$$[R_j^{*2}S_x^2-R_j^2S_x^2]\le 0$$
or,
$$R_j^*\le\pm R_j,(j=18,19,\dots,26) \qquad (14)$$

(viii) Estimator t_j^* is better than sample mean, t_j ([GY17]) whenever:
$$MSE(t_j^*)-MSE(t_j)\le 0$$
or,
$$[R_j^{*2}S_x^2-R_j^2S_x^2]\le 0$$
or,
$$R_j^*\le\pm R_j,(j=27,28,29) \qquad (15)$$

5. EMPIRICAL STUDY

Here, we also adopted the same data set used by Gupta and Yadav ([GY17]) captured from Kadilar and Cingi ([KC04, KC06]) for better judgement on these newly Ratio Type Estimators. These parameters were:

$N = 106, n = 40, \bar{Y} = 2212.59, \bar{X} = 27421.70, \rho = 0.860, S_y = 11551.53, C_y = 5.22$
$S_x = 57460.61, C_x = 2.10, \beta_1 = 2.122, \beta_2 = 34.574, M_d = 7297.50, QD = 12156.25$
$G = 40201.69, D = 35634.99, S_{pw} = 35298.81.$

6. RESULTS AND DISCUSSION

The results obtained are presented in Table 2 below:

Table 2. Mean Squared Errors obtained on the newly modified Ratio Type Estimators and other existing ones considered

Estimator	Constant	Bias	Mean Square Error
t_0	Nil	0	2077627.25
t_r	0.0807	171.32	984589.70
t_1	0.0807	151.20	889617.50
t_2	0.0807	151.18	889566.40
t_2^*	0.08044	150.305	873440.3
t_3	0.0806	150.82	888775.70
t_3^*	0.07681	137.056	844124.0
t_4	0.0807	151.20	889616.00
t_4^*	0.08068	151.20	875421.60
t_5	0.0806	151.02	889215.30
t_5^*	0.07880	144.218	859971.20
t_6	0.0807	151.19	889596.60
t_6^*	0.08059	150.8488	874643.00
t_7	0.0807	151.20	889607.50
t_7^*	0.08059	151.047	875081.50
t_8	0.0807	151.17	889557.80
t_8^*	0.08040	150.156	873109.90
t_9	0.0867	151.20	889616.90
t_9^*	0.08068	151.20	875456.60
t_{10}	0.0806	150.76	888634.40
t_{10}^*	0.07622	134.938	839439.00
t_{11}	0.0807	151.14	889452.50
t_{11}^*	0.08044	150.296	873419.00
t_{12}	0.0807	151.13	889492.90
t_{12}^*	0.07881	144.288	860126.40
t_{13}	0.0637	94.32	763783.60
t_{13}^*	0.00693	1.11523	543344.00
t_{14}	0.0715	119.04	818477.40
t_{14}^*	0.01330	4.105817	549960.90
t_{15}	0.0767	136.64	857402.20
t_{15}^*	0.01341	4.17786	550120.30
t_{16}	0.0801	148.10	884526.80
t_{16}^*	0.06169	88.40545	736481.50
t_{17}	0.0742	128.08	838466.80
t_{17}^*	0.00862	1.72748	544698.70
t_{18}	0.0327	24.87	610126.10
t_{18}^*	0.00135	0.04251	540970.50
t_{19}	0.0475	52.34	670914.00
t_{19}^*	0.00117	0.03159	540946.40
t_{20}	0.0297	20.59	600579.70
t_{20}^*	0.00279	0.18075	541276.40
t_{21}	0.0320	23.85	607875.10
t_{21}^*	0.00152	0.05388	540995.7
t_{22}	0.0498	57.60	682552.70
t_{22}^*	0.00131	0.04006	540965.10
t_{23}	0.0351	28.59	618381.50
t_{23}^*	0.00313	0.22803	541381.00
t_{24}	0.0322	24.12	608480.30
t_{24}^*	0.00154	0.05489	540997.90
t_{25}	0.0500	58.02	683478.00
t_{25}^*	0.00133	0.04081	540966.80
t_{26}	0.0353	28.90	619061.50
t_{26}^*	0.00316	0.23222	541390.30
t_{27}	0.00245	0.062	565007.83
t_{27}^*	0.00055	0.00712	540892.20
t_{28}	0.00262	0.064	565132.91
t_{28}^*	0.00062	0.00905	540896.50
t_{29}	0.00273	0.067	565334.48
t_{29}^*	0.00063	0.00922	540896.90

Here, these newly modified ratio type estimators are more efficient than their respective Gupta and Yadav ([GY17]) existing ratio type estimators for estimating population mean. Consequently, the newly modified ratio type estimator, t_{27}^* , is the best since it has the smallest mean squared error, hence most efficient and preferred.

7. CONCLUSION

It has been found and established that these newly modified ratio type estimators are more efficient than earlier Gupta and Yadav ([GY17]) existing ratio type estimators. They are respectively more efficient but the newly modified ratio type estimator, t_{27}^* , is the best since it has the smallest mean squared error. Hence, it is recommended for usage in sampling.

REFERENCES

[A+16] **M. Abid, N. Abbas, R. A. K. Sherwani, H. Z. Nazir** – *Improved Ratio Estimators for the Population Mean Using Non-Conventional Measures of Dispersion*. Pakistan Journal of Statistics and Operations Research, 12, pp. 353-367, 2016.

[Coc40] **W. G. Cochran** – *Sampling Techniques*. 1st Edition, Wiley Eastern Limited, New Delhi, 1940.

[Coc77] **W. G. Cochran** – *Sampling Techniques*. 3rd Edition, New York: John and Sons, New Delhi, 1977.

[GY17] **R. K. Gupta, S. K. Yadav.** – *New Efficient Estimators of Population Mean Using Non-Traditional Measures of Dispersion*. Open Journal of Statistics 7, pp. 394-404, 2017.

[JMM13] **M. I. Jeelani, S. Maqbool, S. A. Mir** – *Modified Ratio Estimators of Population Mean Using Linear Combination of Coefficient of Skewness and Quartile Deviation*. International Journal of Modern Mathematical Sciences, 6, pp. 174-183, 2013.

[KC04] **C. Kadilar, H. Cingi** – *Ratio Estimators in Simple Random Sampling*. Applied Mathematics and Computation, 151, pp. 893-902, 2004.

[KC06] **C. Kadilar, H. Cingi** - *An Improvement in Estimating the Population Mean by Using the Correlation Coefficient*. Hacettepe Journal of Mathematics and Statistics, 35, pp. 103-109, 2006.

[SJ12] **J. Subramani, G. Kumarapandiyan** – *Estimation of Population Mean Using Co-Efficient of Variation and Median of an Auxiliary Variable*. International Journal of Probability and Statistics, 1, pp. 111-118, 2012.

[YT10] **Z. Yan, B. Tian** – *Ratio Method to the Mean Estimation Using Coefficient of Skewness of Auxiliary Variable*. In: Zhu, R., Zhang, Y., Liu, B. and Liu, C., Eds., Information Computing and Applications. ICICA 2010. Communications in Computer and Information Science, 106, Springer, Berlin, Heidelberg, pp. 103-110, 2010.